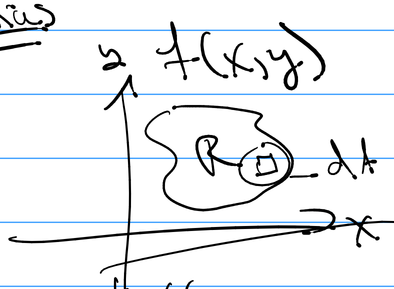
Math 344

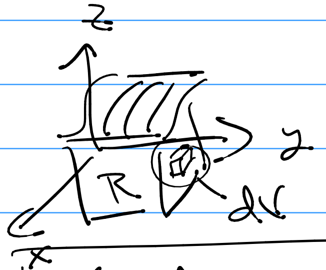
Exam 3 11 probs @ 10pts each 100pts = 100%

Ch15 $f : \mathbb{R}^{\overset{2,3}{n}} \to \mathbb{R}$ $f(x,y)$ or $f(x,y,z)$

Integrations



$$\iint_R f\,dA$$



$$\iiint_R f\,dV$$

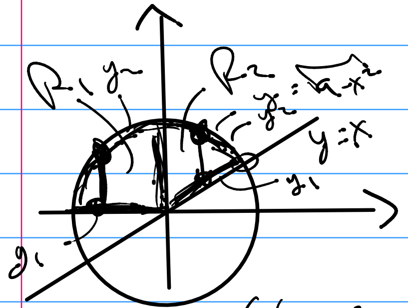
15.1/15.2 $\iint_R f\,dA$ in cartesian $dx\,dy$

2 probs ① R is rectangular

② R is not rectangular

ex3 integrate $f(x,y) = x^3 + \sin y$ over region bounded by upper half of circle centered @ origin of radius 3, x-axis, and above $y = x$

$x^2+y^2=9$



$$\iint_R (x^3+\sin y)\,dA$$

$$R = R_1 \cup R_2$$

$x^2+x^2=9$

$2x^2=9$

$$\iint_{R_1} x^3+\sin y\,dA + \iint_{R_2} x^3+\sin y\,dA$$

$$= \int_{-3}^{0}\int_{0}^{\sqrt{9-x^2}} \underline{x^3+\sin^2 y}\, dy\, dx + \int_{0}^{3/\sqrt{2}}\int_{x}^{\sqrt{9-x^2}} x^3+\sin^2 y\, dy\, dx$$

$= \textcircled{?}$

15.3 Polar

① $\iint_{R} f\, dA$

R — r, θ's

f — function of r, θ's

dA — $r\,dr\,d\theta$

Use

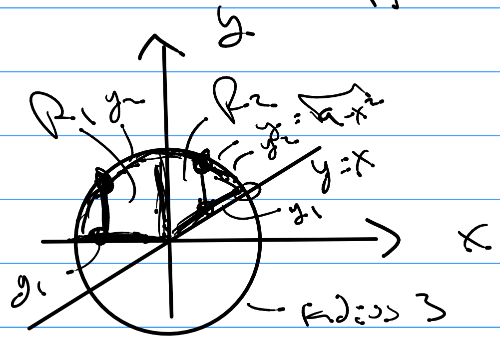
$x = r\cos\theta$

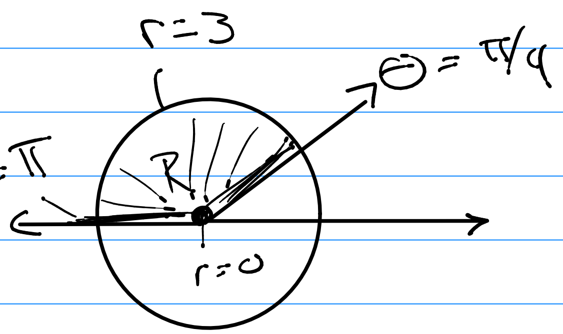
$y = r\sin\theta$

$x^2+y^2 = r^2$

$\tan\theta = y/x$

ex





$$\iint_{R} (\underbrace{x^2+y^2}_{} + 3x)\, dA = \int_{\pi/4}^{\pi}\int_{0}^{3} \textcircled{f}\; r\,dr\,d\theta$$

in r, θ's

$$= \int_{\pi/4}^{\pi}\int_{0}^{3} (r^2 + 3r\cos\theta)\, r\,dr\,d\theta$$

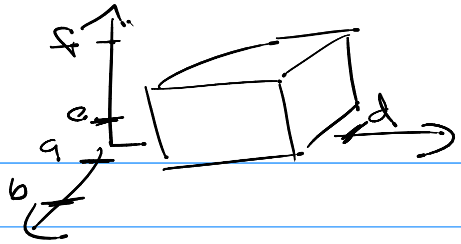
15.4 | 15.5 Applications of $\iint_{R} f\, dA$

2 probs ① Center of Mass

② Surface Area $SA = \iint_{R} \sqrt{1+(f_x)^2+(f_y)^2}\, dA$

15.6 $\iiint_{R} f\, dV$ cartesian (2 probs)

① R is <u>box</u>



$$\iiint f\,dV = \int_a^b \int_c^d \int_e^f f(x,y,z)\,dz\,dy\,dx$$

ⓔⓧ $0 \le x \le 2$, $1 \le y \le 3$, $-1 \le z \le 1$

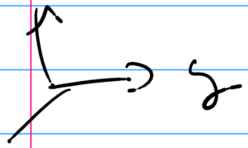
$$\iiint_R (xy+z^2)\,dV = \int_0^2 \left[\int_{-1}^{1} \left[\int_1^3 (xy+z^2)\,dy\right] dz\right] dx$$

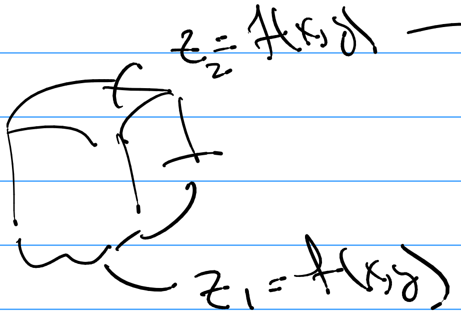
$$\frac{1}{2}xy^2 + z^2y \Big|_1^3 = \frac{9}{2}x + 3z^2 - \frac{1}{2}x - z^2$$

$$= 4x + 2z^2$$

$$= \int_0^2\int_0^1 \underline{(4x+2z^2)}\,dz\,dx$$

② R is <u>not</u> a box?





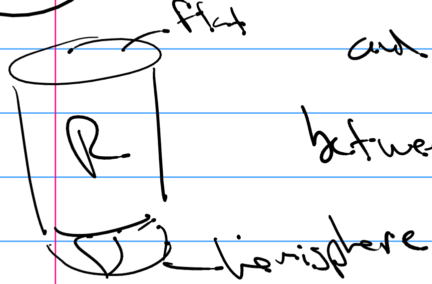
will have a "natural" top bottom

$$\iiint_R f\,dV = \left[\int_{z_1}^{z_2} f\,dz\right]$$

ⓔⓧ R is above $z = \boxed{-\sqrt{1-x^2-y^2}}$

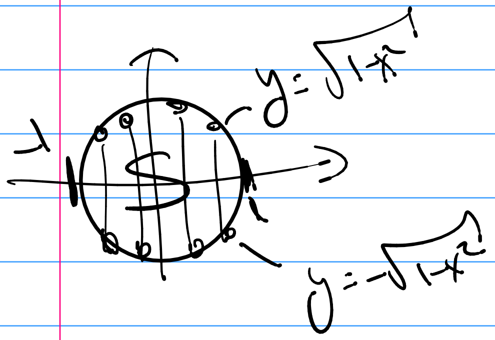
and below $z = \boxed{5}$

between a circle of radius 1 centered @ origin



$$\iiint_R f\,dV = \iint_{\textcircled{S}} \left[\int_{-\sqrt{1-x^2-y^2}}^{5} f\,dz\right] dA$$

xy-plane



$$= \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{-\sqrt{1-x^2-y^2}}^{5} \boxed{f}\,dz\,dy\,dx$$

15.7 $\iiint_R f\,dV$ cylindrical $\textcircled{\text{1 prb}}$

15.8 $\iiint_R f\,dV$ spherical $\textcircled{\text{1 prb}}$

$\boxed{15.9}$ Change of Variables $\textcircled{\text{2 prb}}$

① 2D
② 3D

$\Big\}$ Setup

$$\iiint_R f\,dV = \iiint_{\textcircled{R}_{u,v,w}} f(u,v,w)\left|\begin{matrix} x_u & x_v & x_w \\ y_u & y_v & y_w \\ z_u & z_v & z_w \end{matrix}\right| du\,dv\,dw$$