Math 344

Calculus

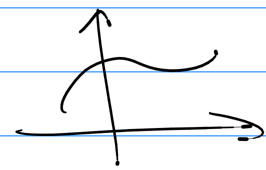
toys = functions f: Domain $\to$ CoDomain

rule

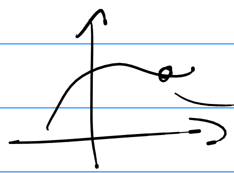
rules = Algebra, Trig, Arith.

limits? , Change? , Sums?

① $f: \mathbb{R} \to \mathbb{R}$



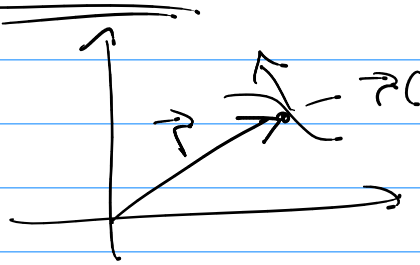
②



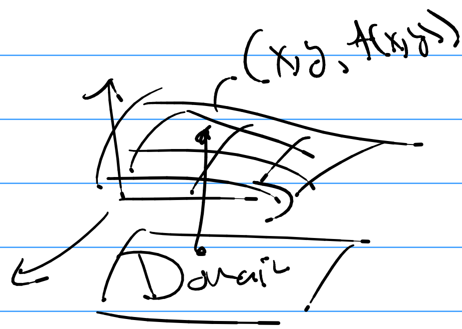
parametric $(x(t), y(t))$

polar

③ Vector Functions: $f: \mathbb{R}^1 \to$ Vectors



④ $f: \mathbb{R}^n \to \mathbb{R}$



$\left(f: \mathbb{R}^2 \to \mathbb{R}, \quad f: \mathbb{R}^3 \to \mathbb{R} \right)$

3D   4D

$\nabla f = \langle f_x, f_y, f_z \rangle$

$\nabla f(x,y,z) = \langle f_x(x,y,z), f_y(x,y,z), f_z(x,y,z) \rangle$

$f: \mathbb{R}^3 \to$ Vector

(x,y,z)

ch16

Domain, CoDomain

$f: \mathbb{R}^n \to \text{Vectors}$

$f: \mathbb{R}^2 \to \text{Vectors}$ $\qquad$ $f: \mathbb{R}^3 \to \text{Vectors}$

Vector Calculus

$f: \mathbb{R}^n \to \text{Vector}$ are normally called Vector fields

$\boxed{\text{Consider how to visualize them}}$

ex $\quad f: \mathbb{R}^2 \to \text{Vector}$

Notation: Vectors $\langle \text{x-coord}, \text{y-coord} \rangle$

$\vec{V} = (\text{x-coord})\,\vec{i} + (\text{y-coord})\,\vec{j}$

$\mathbf{V}$ in bold font

Functions: $$\mathbf{f}(x,y,z) = \langle \underbrace{f_1(x,y,z)}_{\text{x-coord of vector}}, \underbrace{f_2(x,y,z)}_{\text{y-coord}}, \underbrace{f_3(x,y,z)}_{\text{z-coord.}} \rangle$$

or

$\vec{f}$

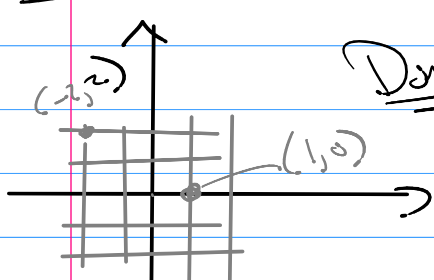
or

$\vec{\mathbf{F}}(x,y,z) = \langle \quad , \quad , \quad \rangle$

Consider: $\mathbf{F}(x,y) = \langle -y, x^2 \rangle$

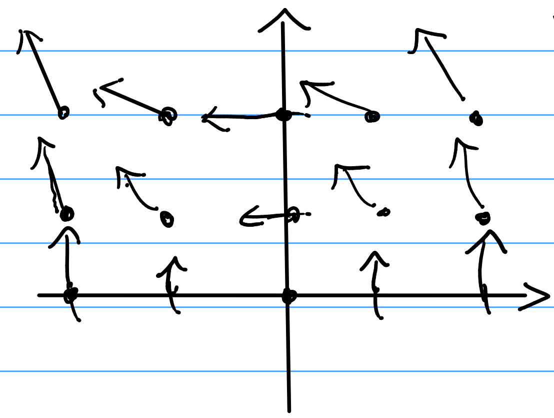
Domain: $\mathbb{R}^2$

input	$\mathbf{F}$
$(-2,2)$	$\langle -2,4 \rangle$
$(-1,2)$	$\langle -2,1 \rangle$
$(0,2)$	$\langle -2,0 \rangle$
$(1,2)$	$\langle -2,1 \rangle$
$(2,2)$	$\langle -2,4 \rangle$
$(-2,1)$	$\langle -1,4 \rangle$



input	F
(-2, 1)	<-1, 4>
(-1, 1)	<-1, 1>
(0, 1)	<-1, 0>
(1, 1)	<-1, 1>
(2, 1)	<-1, 4>
(-2, 0)	<0, 4>
(-1, 0)	<0, 1>
(0, 0)	<0, 0>
(1, 0)	<0, 1>
(2, 0)	<0, 4>
	⋮

$\mathbf{F}(x,y) = \langle -y, x^2 \rangle$



Vector Fields

Note:

$$\mathbf{F}(x,y) = \langle \underbrace{f_1(x,y)}, \underbrace{f_2(x,y)} \rangle$$

Scalar functions

$$\mathbf{F}(x,y,z) = \langle f_1(x,y,z), f_2(x,y,z), f_3(x,y,z) \rangle$$

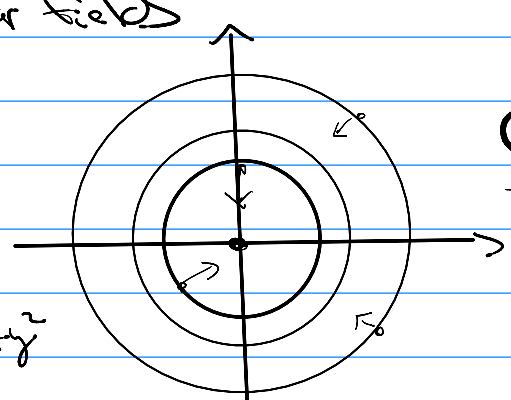
b/c $\mathbf{F}$ is a vector we now get.

1. Dot products
2. Cross products
3. Magnitude
4. gradients is of a scalar function gives a vector field.

Why?

1. gradient vector fields
2. gravity

$$|\text{Force}| = \frac{gMm}{r^2} \quad r^2 = x^2 + y^2$$



$\mathbf{G}(x,y) = ?$