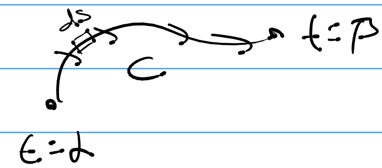
# Math 344

$\int_C f \, ds$

scalar function.




$C: x(t), y(t), z(t)$

$\vec{r} = \langle x(t), y(t), z(t) \rangle$

$$\int_C f\,ds = \int_\alpha^\beta f(\vec{r}) \underbrace{\boxed{|\vec{r}\,'|\,dt}}_{ds}$$

$$= \int_\alpha^\beta f(x(t), y(t), z(t)) \sqrt{(x')^2 + (y')^2 + (z')^2}\, dt$$

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wrt $x$ $\int_C f\,dx$, $dx = x'\,dt$ wrt $y$ $\int_C f\,dy$, $dy = y'\,dt$ wrt $z$ $\int_C f\,dz$, $dz = z'\,dt$

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Notation: $\int_C P\,dx + \int_C Q\,dy + \int_C R\,dz$

$$= \int_C P\,dx + Q\,dy + R\,dz$$

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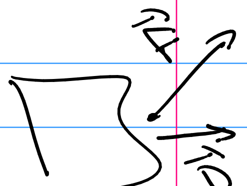
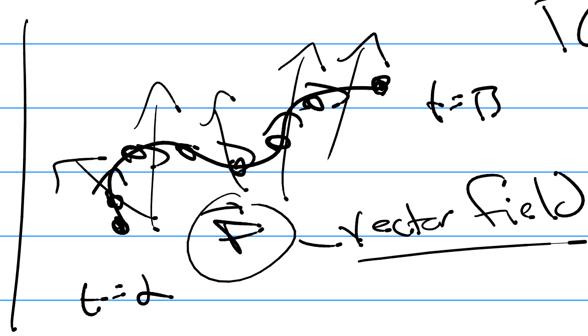
Application Work = (Mag. of force) (displacement) in the direction.

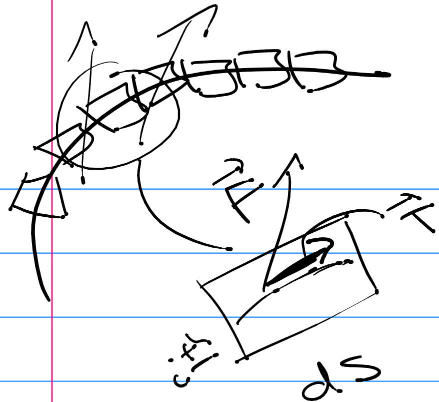



$W = \vec{F} \cdot \vec{D}$




Q: Work by $\vec{F}$ on particle moving on C

???




$$w_i = (\vec{F}_i) \cdot (ds\, \vec{T}_i)$$

$$w_i = \underbrace{(\vec{F}_i \cdot \vec{T}_i)}_{\text{scalar function.}} ds$$

$$W = \int_C \underbrace{(\vec{F} \cdot \vec{T})} ds$$

Note: $\vec{F} \cdot \vec{T}$ = scalar function.

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Notation $\quad W = \int_C (\vec{F} \cdot \vec{T}) ds$

but: $\quad \vec{T} = \dfrac{\vec{r}\,'}{|\vec{r}\,'|} \qquad ds = |\vec{r}\,'| dt$

$$W = \int_C (\vec{F} \cdot \vec{T}) ds = \int_\alpha^\beta (\vec{F} \cdot \vec{r}\,') dt$$

Ex $\vec{F} = \langle 2x, y, 3z \rangle$ along C of
$x(t) = 3t^2 + 1$
$y(t) = t - 2$
$z(t) = 1/t$
from $t = 1$ to $t = 2$

$$W = \int_C (\vec{F} \cdot \vec{T}) ds = \int_1^2 \langle 2(3t^2+1), t-2, \tfrac{3}{t} \rangle \cdot \langle 6t, 1, \tfrac{-1}{t^2} \rangle dt$$

$$= \int_1^2 (36t^3 + 12t^2) + (t-2) - \frac{3}{t^3} \, dt \;\boxed{\text{Work}}$$

One more notation change ...

$$ds = \sqrt{(x')^2+(y')^2+(z')^2}\,dt$$

b/c $\underline{dx = (x')dt}$ , $\underline{dy = y'dt}$ , $\underline{dz = z'dt}$

$$\text{Work} = \int_C (\vec{F}\cdot\vec{T})\,ds = \int_\alpha^\beta \vec{F}\cdot\vec{r}\,'\,dt = \boxed{\int_C \vec{F}\cdot d\vec{r}}$$

$$d\vec{r} = \vec{r}\,'\,dt$$

if $\vec{F} = \langle P, Q, R\rangle$ $\vec{T}dt = \langle x', y', z'\rangle dt$

$$\text{Work} = \int_C (\vec{F}\cdot\vec{T})\,ds = \int_C \vec{F}\cdot d\vec{r} = \int_C P\,dx + Q\,dy + R\,dz$$

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if $f$ and $\vec{F} = \nabla f$ we call $\vec{F}$ a conservative vector field.

given $\vec{F} = \langle P, Q, R\rangle$

find that $f$ exists such that $P = f_x$
$Q = f_y$
$R = f_z$

$f$ is the potential function.

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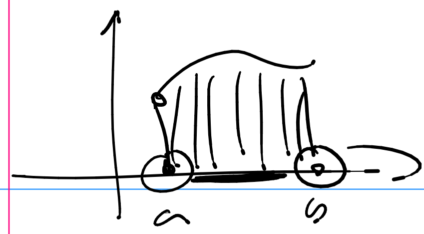
$$\int_a^b \left(\frac{d}{dx}[f(x)]\right)dx = f(b) - f(a)$$

know $\int \frac{d}{dx}\{f(x)\}\,dx = f(x) + C$

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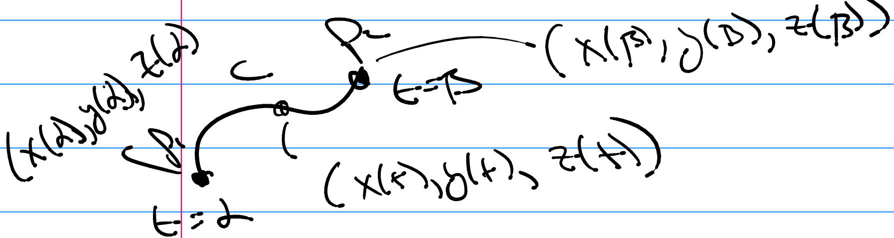
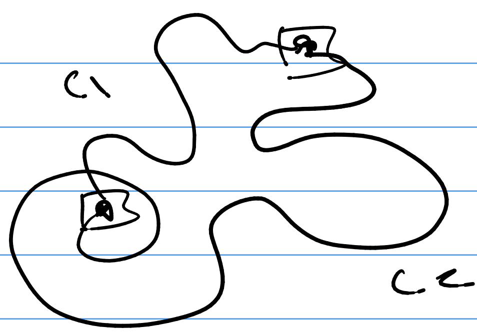
## Fund. thm for line integrals

Normal Fund. thm $\displaystyle\int_a^b \frac{d}{dx}[f(x)]\,dx = f(b) - f(a)$

$$\int_C \nabla f \cdot \underbrace{d\vec{r}}_{\vec{r}\,'\,dt} = \int_\alpha^\beta \nabla f \cdot \vec{r}\,'\,dt = f(\vec{r}(\beta)) - f(\vec{r}(\alpha))$$

$$= f(P_2) - f(P_1)$$




$$\int_{C_1} \nabla f \cdot d\vec{r} = \int_{C_2} \nabla f \cdot d\vec{r}$$

path independence.

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**Big Q** $\displaystyle\int_C \vec{F} \cdot d\vec{r} = ?$

if $\boxed{\vec{F} = \nabla f}$ then $f(\text{end pt}) - f(\text{start point})$

$\vec{F} = \langle P, Q, R \rangle \overset{?}{=} \langle f_x, f_y, f_z \rangle \rightarrow f = ?$

(1) existence of $f$ for $\nabla f = \vec{F}$

→ Independence of Path.

if $\oint_C \vec{F} \cdot d\vec{r} = 0$ for any closed path $C$

then $\vec{F}$ is conservative.

( start looking for $f$ such that $\nabla f = \vec{F}$ )