Math 344

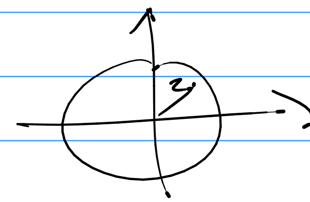
Q's

Exam paraboloid $\cancel{z} = x^2+y^2$ $z=4$ → polar

$$SA = \iint_R \sqrt{1+f_x^2+f_y^2}\,dA$$

$f_x = 2x$

$f_y = 2y$

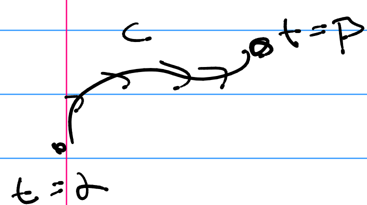


$4 = x^2+y^2$

$$SA = \iint_R \sqrt{1+4x^2+4y^2}\,dA$$

$$SA = \int_0^{2\pi}\int_0^2 \sqrt{1+4r^2}\,r\,dr\,d\theta$$

Line Integrals



$$\int_C (f)\,ds$$

vs $\int_C (f)\,dx$, $\int_C (f)\,dy$,

Notation: typical application was work. $\vec{r}$ is C

2D

$$\text{Work} = \int_C (\vec{F}\cdot\vec{T})\,ds = \int_\alpha^\beta \vec{F}(\vec{r})\cdot\vec{r}\,'\,dt$$

$\vec{F} = \langle P, Q\rangle$

$$= \int_C \vec{F}\cdot d\vec{r}$$

$$= \int_C P\,dx + Q\,dy$$

want to find $\int_C \vec{F}\cdot d\vec{r} = ?$

$\boxed{\text{Fund. thm}}$ $\int_C \nabla f \cdot d\vec{r} = f(x_2,y_2) - f(x_1,y_1)$

2D $\boxed{\vec{F} = \nabla f}$ → says $\vec{F} = \langle P, Q\rangle = \langle f_x, f_y\rangle$ for some f

call $\vec{F}$ conservative
f potential function

$\boxed{Q}$ to use Fund. thm to calculate $\int_C \vec{F}\cdot d\vec{r}$

is $\vec{F} = \nabla f$? yes → find f

$\boxed{\text{When is } \vec{F} = \nabla f}$

1^{st} path ind. if and only if $\int_C \vec{F}\cdot d\vec{r} = 0$

thm 3,4 if and only if $\vec{F} = \nabla f$.

ch14 if f is defined on a region and f_x, f_y
f_{xx}, f_{yy}
f_{xy}, f_{yx}

if f_x, f_y are cont on region
then $f_{xy} = f_{yx}$

ex

ch14

$f(x,y) = x^2y + y^3x$
$f_x = \boxed{2xy + y^3}$ $f_y = \boxed{x^2 + 3y^2x}$
$f_{xy} = 2x + 3y^2 = f_{yx} = 2x + 3y^2$

thm $\vec{F} = \langle P, Q \rangle$ want $\langle P, Q \rangle = \nabla f = \langle f_x, f_y \rangle$

$$P = f_x \qquad Q = f_y$$

check $P_y = f_{xy}$ $\quad Q_x = f_{yx}$

if equal then $\vec{F} = \nabla f$

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ex $\vec{F} = \langle 3xy + y^2, 6y + x^3 \rangle$

$$f_x = P = 3xy + y^2 \qquad f_y = Q = 6y + x^3$$

check mixed partials
$$\left.\begin{aligned} f_{xy} = P_y &= 3x + 2y \\ f_{yx} = Q_x &= 3x^2 \end{aligned}\right\} \underline{\text{not equal!}}$$

So $\vec{F}$ is not conservative, so can not use fund. thm.
(not path independent)

ex $\vec{F} = \langle \sin(x) + xy, \cos(y) + \frac{1}{2}x^2 \rangle$

$$P = \sin(x) + xy \qquad Q = \cos(y) + \tfrac{1}{2}x^2$$

check: $P_y = x \qquad Q_x = x$

b/c $P_y = Q_x \rightarrow \vec{F} = \langle f_x, f_y \rangle = \nabla f$

Find f?

$$\text{ex} \quad \langle \overset{f_x}{\boxed{\sin(x)+xy}}, \overset{f_y}{\boxed{\cos(y)+\tfrac{1}{2}x^2}} \rangle = \nabla f \rightarrow f = ?$$

<u>know:</u> $f_x = \sin(x) + xy$

$$\frac{\partial}{\partial x}[f] = \sin(x) + xy$$

constant to ∂x

$$f = \int \sin(x) + (xy)\, dx = -\cos(x) + \tfrac{1}{2}x^2 y + C$$

$C(y)$

So $f = -\cos x + \tfrac{1}{2}x^2 y + C(y)$

$$f_y = 0 + \tfrac{1}{2}x^2 + C'(y)$$

<u>now compare to above</u> $f_y = \cos(y) + \tfrac{1}{2}x^2$

So $C'(y) = \cos(y)$

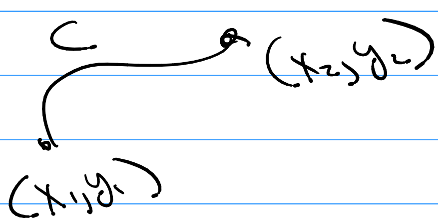
$$C(y) = \int \cos(y)\,dy = \sin(y) + C$$

$$\boxed{f(x,y) = -\cos x + \tfrac{1}{2}x^2 y + \sin(y) + C}$$

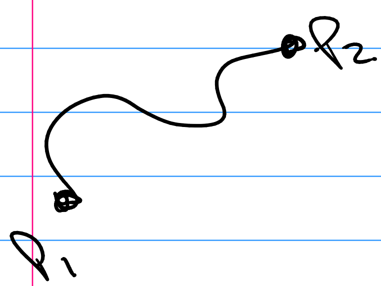
<u>check</u> $f_x = \sin x + xy$ $\rightarrow$ this is our $\vec{F}$

$f_y = \cos y + \tfrac{1}{2}x^2$

$$\int_C \vec{F} \cdot d\vec{r} = f(x_2,y_2) - f(x_1,y_1)$$



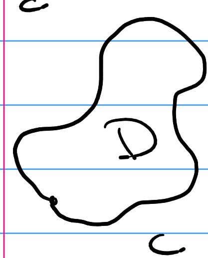
$\boxed{\text{So}}$ $\displaystyle\int_C \vec{F}\cdot d\vec{r} = \int_C (\vec{F}\cdot\vec{T})\,ds$ = **work.**



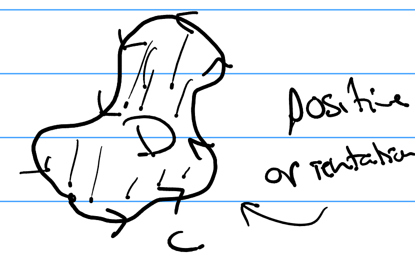
If $\vec{F} = \nabla f$

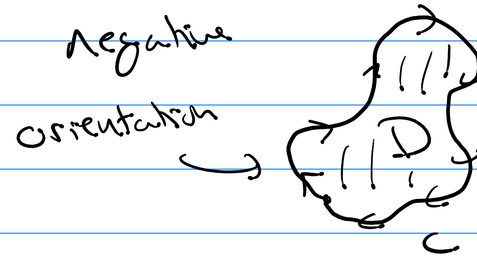
use **fund. thm** : work $= f(P_2) - f(P_1)$

$\displaystyle\int_C \vec{F}\cdot d\vec{r}$ on non-conserv. vector fields



$\displaystyle\int_C \vec{F}\cdot d\vec{r} = ?$





$\boxed{\text{Green's thm}}$ C is a positively oriented, piecewise smooth, simple closed curve that bounds region D.

and $\vec{F} = \langle P, Q\rangle$ that has cont. partials on an open region containing D

then $\displaystyle\int_C \vec{F}\cdot d\vec{r} = \boxed{\int_C P\,dx + Q\,dy} = \boxed{\iint_D (Q_x - P_y)\,dA}$

Note: easily show $\displaystyle\int_C \vec{F}\cdot d\vec{r} = 0$ for $\underbrace{\text{conserv. vector fields}}_{\vec{F} = \langle P, Q\rangle = \nabla f}$

$\rightarrow P_y = Q_x$