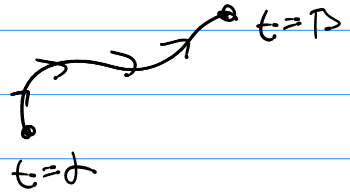
Math 344

Vector fields $F = \langle P, Q \rangle$ ⓶D $\quad F = \langle P, Q, R \rangle$ ③D

$$\int_C f\,ds = \int_\alpha^\beta f(\vec{r})|\vec{r}|dt$$



Specific form

$$\int_C \vec{F}\cdot\vec{T}\,ds = \int_\alpha^\beta \vec{F}(\vec{r})\cdot\vec{r}\,'\,dt$$

$$= \int_C \vec{F}\cdot d\vec{r}$$

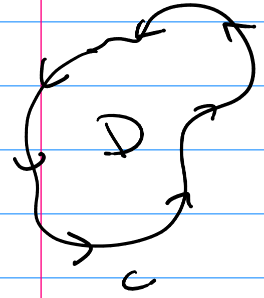
$$= \int_C P\,dx + Q\,dy + R\,dz$$

Note (#1) (if) $\vec{F} = \nabla f$

we can use Fund Thm $\displaystyle\int_C \vec{F}\cdot d\vec{r} = f(\overset{\text{end pt}}{\boxed{\vec{r}(\beta)}}) - f(\overset{\text{start pt}}{\boxed{\vec{r}(\alpha)}})$

and $\displaystyle\int_C \vec{F}\cdot d\vec{r} = 0$ if C is a closed curve.

(#2) for an $\vec{F} = \langle P, Q \rangle$ (2D)



Green's thm

$$\int_C \vec{F}\cdot d\vec{r} = \iint_D (Q_x - P_y)\,dA$$

Notation

boundary is closed

$$\int_C \to \int_{\partial D} \to \oint_C$$

↑ is the boundary of D = ∂D

↑ boundary

typical notation:

$\vec F = \langle P, Q \rangle$

$$\oint_C \vec F \cdot d\vec r = \boxed{\begin{aligned}&\oint_C P\,dx + Q\,dy \\ &= \iint_D (Q_x - P_y)\,dA\end{aligned}}$$

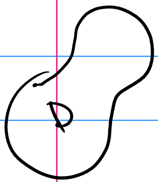
Green's thm →

$$\boxed{\begin{gathered}\iint_D (Q_x - P_y)\,dA = \oint_{\partial D} P\,dx + Q\,dy \\ \oint_{\partial D} P\,dx + Q\,dy = \iint_D (Q_x - P_y)\,dA\end{gathered}}$$

Use?

ex work for $\vec F = \langle P, Q \rangle$ non-conserv. closed curve C

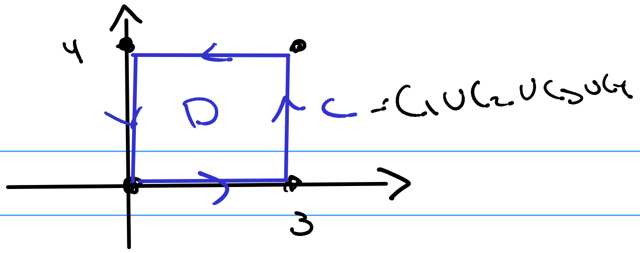
$$\oint_C \vec F \cdot d\vec r = \oint_C P\,dx + Q\,dy$$



use green's $= \iint_D (Q_x - P_y)\,dA$

$\vec{F} = \langle ye^x, 2e^x \rangle$

ex $\int_C ye^x\,dx + 2e^x\,dy$



greens → $= \iint_D (2e^x - e^x)\,dA$

$= \iint_D e^x\,dA = \int_0^4\int_0^3 e^x\,dx\,dy$

$= \int_0^4 dy \int_0^3 e^x\,dx = \boxed{(4)(e^3-1)}$

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use?: $\iint_D (Q_x - P_y)\,dA = \oint_{\partial D} P\,dx + Q\,dy$

ex area D $= \iint_D (1)\,dA = ?$

so $Q_x - P_y = 1 \longrightarrow ?$ $\vec{F} = \langle P, Q \rangle$?

① $\boxed{\langle 0, x \rangle}$

② $\langle -y, 0 \rangle$

③ $\langle -\frac{1}{2}y, \frac{1}{2}x \rangle$

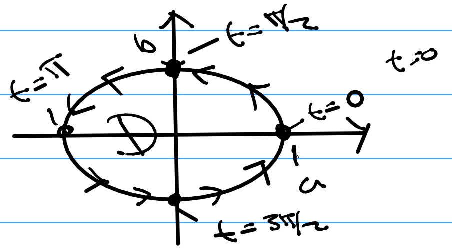
area D $= \iint_D (1)\,dA \overset{①}{=} \boxed{\oint_{\partial D} x\,dy}$

need ∂D $x(t)$

$y(t) \Rightarrow dy = y'\,dt$

ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



$x(t) = a\cos(t)$

$y(t) = b\sin(t) \Rightarrow dy = y'\,dt$

$$\iint_D (1)\,dy\,dx = \int_{-a}^{a}\int_{-\sqrt{(1-\frac{x^2}{a^2})b^2}}^{\sqrt{(1-x^2/a^2)b^2}} (1)\,dy\,dx = \underline{\underline{\text{area}}}$$

greens

$$\oint_{\partial D} x\,dy = \oint_{\partial D} a\cos(t)\,b\cos(t)\,dt$$

$$= \int_0^{2\pi} ab\cos^2(t)\,dt$$

$\boxed{16.5}$ Curl and Divergence of $\vec{F} = \langle P, Q, R\rangle$ (vector field)

ⓔⓧ (physical example) Fluid's (or gasses)

want two operators $\quad op(\vec{F}) = \boxed{\text{output}}$

① Curl$(\vec{F})$ a) Vector where fluid rotates around that vector

b) mag to be proportional to "speed" of rotation

c) $= \vec{0}$ No rotation (irrotational)

$F = \langle P, Q, R\rangle$

$$\text{curl}(\vec{F}) = \langle R_y - Q_z,\ P_z - R_x,\ Q_x - P_y\rangle$$

Note: $\nabla f = \langle f_x, f_y, f_z\rangle$

$$= \begin{array}{r} \\ \nabla \to \\ \vec{F} \to \end{array} \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = \nabla \times \vec{F}$$

$$\nabla = \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \rangle$$

$$\boxed{\text{curl}(\vec{F}) = \nabla \times \vec{F}}$$

② Div: $\nabla \cdot \vec{F} = P_x + Q_y + R_z$