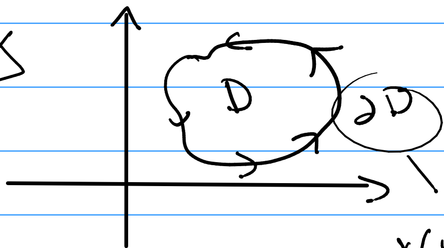
Math 344

Q's

#22



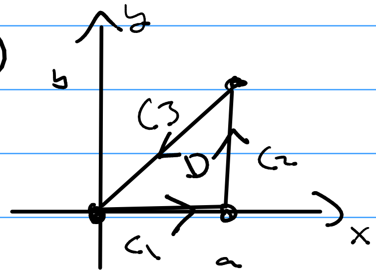
centroid $\bar{x} = \frac{1}{2A}\oint_{\partial D} x^2\,dy$

$\bar{y} = \frac{-1}{2A}\oint_{\partial D} y^2\,dx$

① $x(t)$, $y(t)$ } parametrize curve

② $dy = y'\,dt$, $dx = x'\,dt$

#24



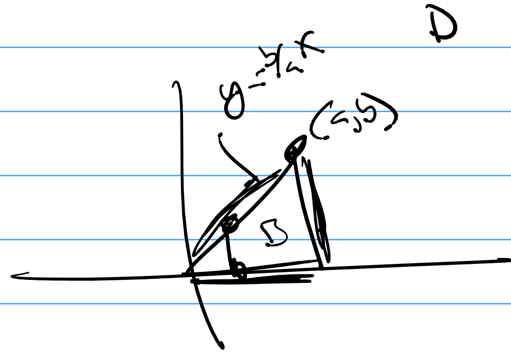
$\bar{x} = \frac{1}{2A}\boxed{\oint_{\partial D} x^2\,dy}$ $\quad \bar{y} = \frac{-1}{2A}\oint_{\partial D} y^2\,dx$

$\partial D = C_1 \cup C_2 \cup C_3$

Area of $D = \frac{1}{2}(a)(b) = A$

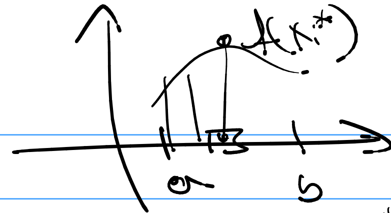
Green's $\oint_{\partial D} (P\,dx + Q\,dy) = \iint_D (Q_x - P_y)\,dA$

$$\boxed{\oint_{\partial D} x^2\,dy} = \oint_{\partial D} (0\,dx + x^2\,dy) \overset{\text{G}}{=} \iint_D (2x - 0)\,dA = \boxed{\iint_D 2x\,dA}$$



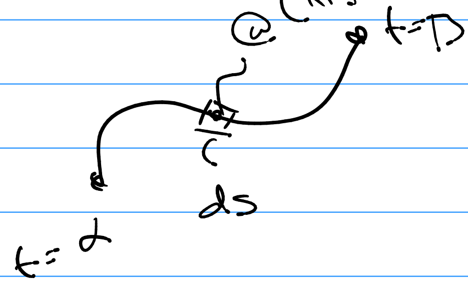
$$\int_0^a \int_0^{\frac{b}{a}x} 2x\,dy\,dx$$

Ch 16 Started calc



$$\int_a^b f\,dx$$

changed it to



$$\int_C f\,ds$$

C ← parametric curve in space

app: work $= \int_C \vec{F}\cdot d\vec{r} = \int_C \vec{F}\cdot\vec{T}\,ds$

find pt: if $\vec{F} = \nabla f \rightarrow \int_C \vec{F}\cdot d\vec{r} = f(\text{end pt}) - f(\text{start pt})$

use: $\text{curl}(\vec{F}) = \nabla \times \vec{F} \overset{?}{=} \vec{0}$

other ops:

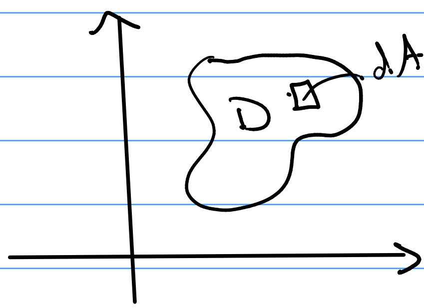
$\text{curl}(\vec{F}) = \nabla \times \vec{F}$

$\text{div}(\vec{F}) = \nabla \cdot \vec{F}$

Laplace operator: $\nabla^2 = \left\langle \frac{\partial^2}{\partial x^2}, \frac{\partial^2}{\partial y^2}, \frac{\partial^2}{\partial z^2} \right\rangle$

$$\nabla^2 = \nabla\cdot\nabla$$

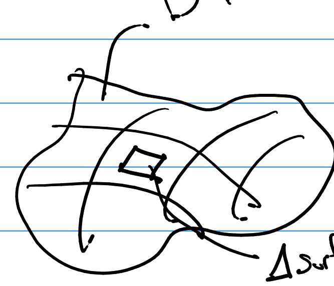
Now



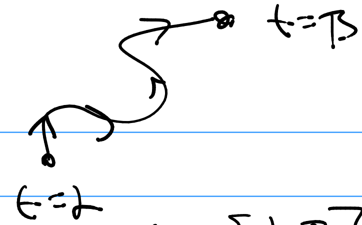
$$\iint_D f\,dA$$

$\xrightarrow[\text{idea}]{\text{modify}}$

D: surface in space

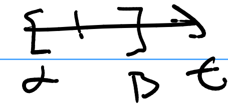


1D object parametric curve

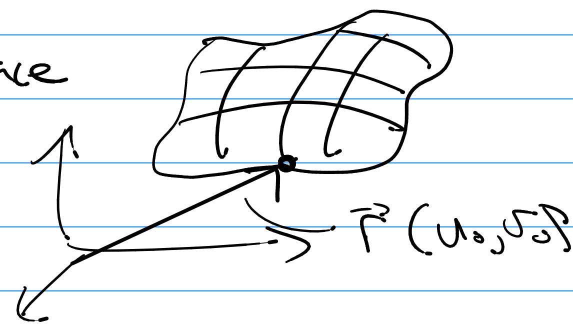


$$\vec{r}(t) = \langle \underbrace{x(t), y(t), z(t)}_{\text{parametric eq'ns}} \rangle$$

$t \in [\alpha, \beta]$



2D object parametric surface



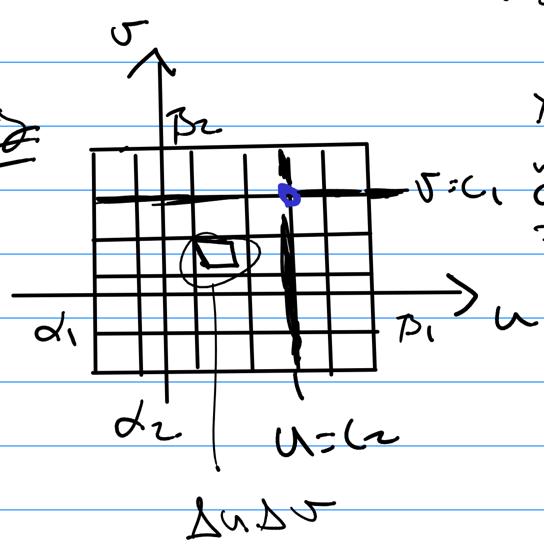
$$\vec{r}(u,v) = \langle \underbrace{x(u,v), y(u,v), z(u,v)} \rangle$$

(1) parametric eq'ns

(2) parameters $u \in [\alpha_1, \beta_1]$

$v \in [\alpha_2, \beta_2]$

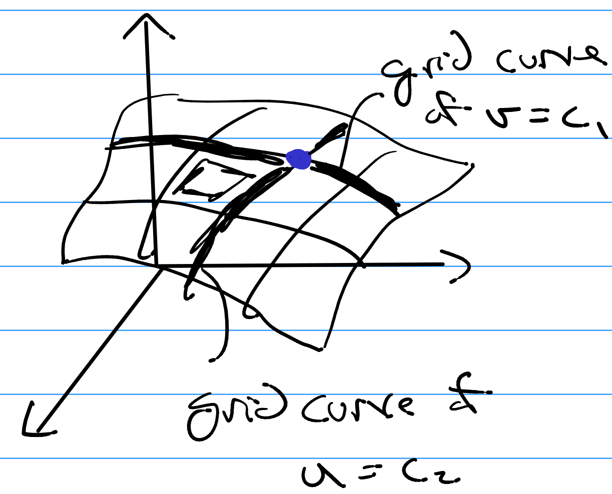
plotting



$x(u,v)$

$y(u,v)$

$z(u,v)$



Ex Surface(,) function of geogebra

$$\text{Surface}(x(u,v), y(u,v), z(u,v), \underbrace{u, \alpha_1, \beta_1}, \underbrace{v, \alpha_2, \beta_2})$$

Quick parameterizations $x(u,v),\ y(u,v),\ z(u,v)$

① Euclidean $z = f(x,y)$

$$\boxed{\begin{aligned} x &= u \\ y &= v \\ z &= f(u,v) \end{aligned}}$$

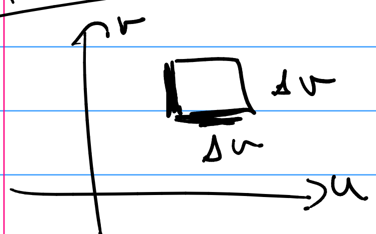
(ex) $z = x^2 + y^2$

$$x = u$$
$$y = v$$
$$z = u^2 + v^2$$

② cylindrical $z = f(r,\theta)$

$$\boxed{\begin{aligned} x &= u \cos v \\ y &= u \sin v \\ z &= f(u,v) \end{aligned}}$$

Application #1 find surface area of parametric surface?



$\vec{r}(u,v) = \langle x(u,v), y(u,v), z(u,v) \rangle$

Δ surface area

$$\Delta \text{ surface area} \approx |\vec{r}_u \Delta u \times \vec{r}_v \Delta v| = |\vec{r}_u \times \vec{r}_v|\, du\, dv$$

$$A(S) = \iint_D |\vec{r}_u \times \vec{r}_v|\, du\, dv$$

D ← in u,v

parametric surface (in u's v's)