Math 344

Q's $\qquad \vec{r} = \langle x, y, z \rangle \qquad \vec{F} = \dfrac{\vec{r}}{|\vec{r}|^p} = \dfrac{1}{(x^2+y^2+z^2)^{p/2}} \langle x, y, z \rangle$

$$\boxed{\begin{aligned} \operatorname{div}(\vec{F}) &= \nabla \cdot \vec{F} = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \cdot \langle P, Q, R \rangle \\ &= P_x + Q_y + R_z \end{aligned}}$$

So $\vec{F} = \dfrac{1}{(x^2+y^2+z^2)^{p/2}} \langle x, y, z \rangle$

$$\vec{F} = \left\langle \boxed{\frac{x}{(x^2+y^2+z^2)^{p/2}}}, \underbrace{\frac{y}{(x^2+y^2+z^2)^{p/2}}}_{Q}, \underbrace{\frac{z}{(x^2+y^2+z^2)^{p/2}}}_{R} \right\rangle$$

$$\operatorname{div}(\vec{F}) = \frac{\partial}{\partial x}\left[\frac{x}{(x^2+y^2+z^2)^{p/2}}\right] + Q_y + R_z$$

$$= \boxed{\frac{(1)(x^2+y^2+z^2)^{p/2} - (x)\left(\frac{p}{2}\right)(x^2+y^2+z^2)^{p/2-1}(2x)}{(x^2+y^2+z^2)^p}} + Q_y + R_z$$

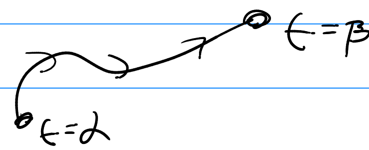
Scratch $\qquad u = x^2+y^2+z^2 \rightsquigarrow \dfrac{u^{p/2} - p x^2 u^{p/2-1}}{u^p}$

$$= \frac{u^{p/2-1}(u - px^2)}{u^p} = \frac{u - px^2}{u^{p/2+1}}$$

$$\operatorname{div}(\vec{F}) = \underline{\underline{\frac{(1-p)x^2+y^2+z^2}{(x^2+y^2+z^2)^{p/2+1}}}} + \underline{Q_y} + \underline{R_z}$$

$$\operatorname{curl}(\vec{F}) = \nabla \times \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}$$

Ch16 parametric curve, C

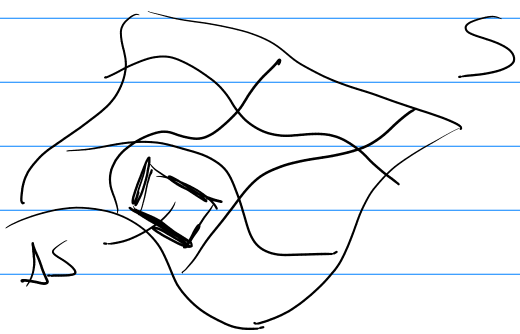


$$\vec{r}(t) = \langle x(t), y(t), z(t) \rangle \qquad t \in [\alpha, \beta]$$

Parametric Surface, S

$$\vec{r}(u,v) = \langle x(u,v), y(u,v), z(u,v) \rangle$$

$$\boxed{\begin{aligned} u &\in [\alpha_1, \beta_1] \\ v &\in [\alpha_2, \beta_2] \end{aligned}}$$

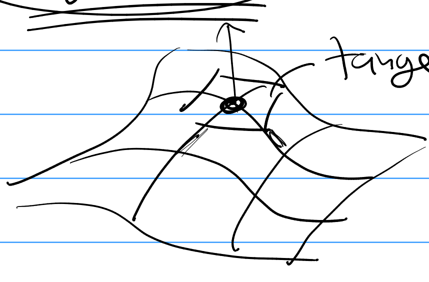


D

Apps (1) Area S, $A(S) = \iint_D |\vec{r}_u \times \vec{r}_v| \, dA$

region of parameters u, v

(2) tangent planes



tangent plane @ $P_0 = \vec{r}(u_0, v_0)$

if $\vec{r}_u \times \vec{r}_v = \vec{0}$ (smooth)

then $\underline{\vec{r}_u} \times \underline{\vec{r}_v}$ is normal to S.

application #3

S of

$x(u,v)$
$y(u,v)$
$z(u,v)$

add up <u>all</u> infinite $f(P^*_{ij})$ on S

16.7 Surface Integral

$$\lim_{m,n\to\infty} \sum_{i=1}^{m} \sum_{j=1}^{n} f(P^*_{ij})\,\Delta S_{ij} = \iint_S f\, dS$$

<u>again:</u> $\underline{dS} = |\vec{r}_u \times \vec{r}_v|\, du\, dv$

$$\iint_S f\, dS = \iint_D f(\vec{r}(u,v))\, |\vec{r}_u \times \vec{r}_v|\, dA$$

region of parameters u,v

Ch 16 New <u>ideas</u> on integration

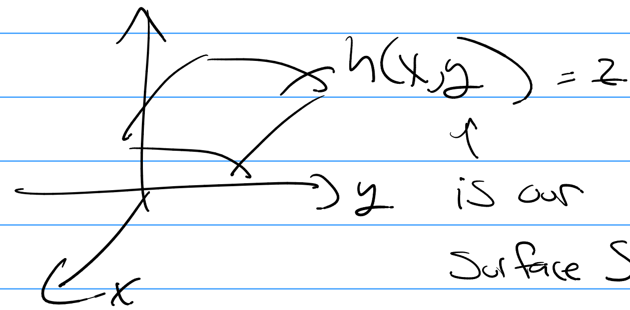
1D $\quad \displaystyle\int_C f\, ds = \int_\alpha^\beta f(\vec{r}(t))\, |\vec{r}\,'|\, dt$

2D $\quad \displaystyle\iint_S f\, dS = \iint_D f(\vec{r}(u,v))\, |\vec{r}_u \times \vec{r}_v|\, dA$

u,v region

Ex given surface in explicit cartesian ...

$$\underline{\underline{z = h(x,y)}}$$



Find $\iint_S f\, dS = \ldots$ need S as
$x(u,v)$
$y(u,v)$
$z(u,v)$

so $\underline{z = h(x,y)} \longrightarrow$ $x = u$, $y = v$, $z = h(u,v)$ is S

so S, $\vec{r}(u,v) = \langle u, v, h(u,v) \rangle$

Now $\iint_S f\, dS = \iint_D f(u, v, h(u,v)) \underline{\underline{|\vec{r}_u \times \vec{r}_v|}}\, dA$

$$\underbrace{|\vec{r}_u \times \vec{r}_v|}_{} = \left\| \begin{vmatrix} i & j & k \\ 1 & 0 & h_u \\ 0 & 1 & h_v \end{vmatrix} \right\| = \sqrt{1 + h_u^2 + h_v^2}$$

$$\boxed{\iint_S f\, dS = \iint_D f(x,y,h(x,y)) \sqrt{1 + h_x^2 + h_y^2}\, dA} \; \star$$

$z = h(x,y)$