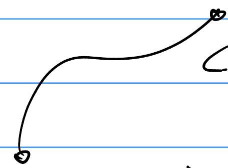
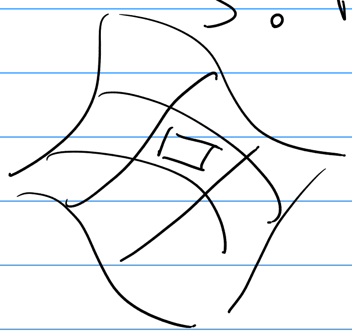
# Math 344

(1D) $\int_C f \, ds = \int_\alpha^\beta f(\vec{r}(t)) \, |\vec{r}'| \, dt$




(2D) $S : \vec{r}(u,v) = \langle x(u,v), y(u,v), z(u,v) \rangle$



$$\iint_S f \, dS = \iint_D f(\vec{r}) \, |\vec{r}_u \times \vec{r}_v| \, dA$$

↑ in $u, v$

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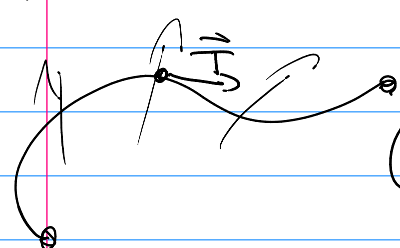
Application similar to line integral and vector fields:

(1D)



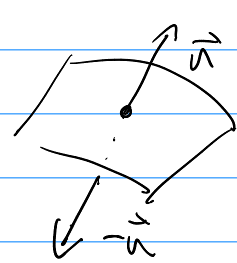

(work) $= \int_C (\vec{F} \cdot \vec{T}) \, ds$

→ Notation $\int_C \vec{F} \cdot d\vec{r} = \int_C P\,dx + Q\,dy + R\,dz$

$$= \int_\alpha^\beta \vec{F}(\vec{r}) \cdot \vec{r}' \, dt$$

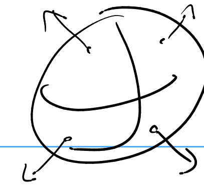
(2D) Surface Integrals and Vector Fields


two sides: orientable surface




$$\vec{n} = \frac{\vec{r}_u \times \vec{r}_v}{|\vec{r}_u \times \vec{r}_v|}$$

given solid region with S as its boundry



positive orientation is outward from solid region

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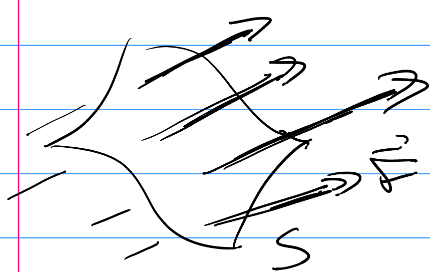
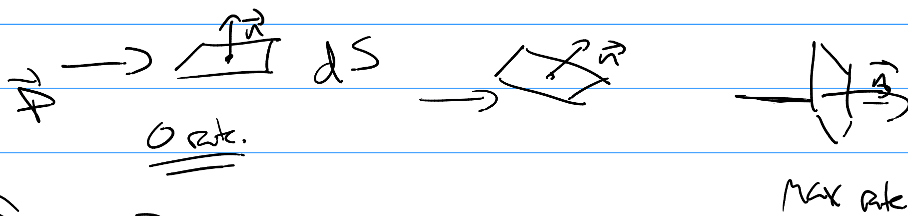
(ex) Application of $\vec{F}$ and S.




$\vec{F}$ is fluid flow through S.

(Q) rate of flow through S?




add up $\vec{F} \cdot \vec{n}$ ← across all S

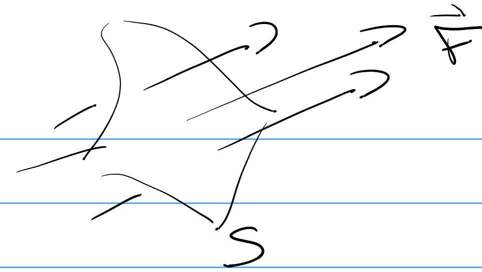
$$\iint_S (\vec{F} \cdot \vec{n})\, dS$$ rate of flow for fluid through S.

Note: real problem to do $\vec{F} = \rho \vec{v}$

($\rho$ = density, $\vec{v}$ = velocity field)

$$\iint_S (\vec{F} \cdot \vec{n})\, dS = \iint_S (\rho \vec{v} \cdot \vec{n})\, dS$$

In general for any $\vec{F}$




$$\iint_S (\vec{F}\cdot\vec{n})\,dS$$ is called flux of $\vec{F}$ across $S$.

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**Notation** $$\iint_S (\vec{F}\cdot\vec{n})\,dS = \iint_S \vec{F}\cdot d\vec{S}$$

$$= \iint_D \left(\vec{F}\cdot(\vec{r}_u\times\vec{r}_v)\right) dA$$

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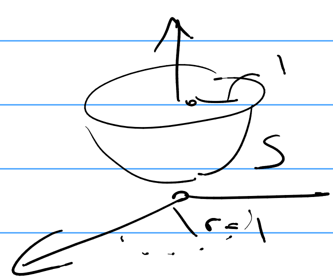
**ex** $\vec{F} = \langle x, y, 1\rangle$ flux across $z = x^2+y^2$ over $x^2+y^2=1$



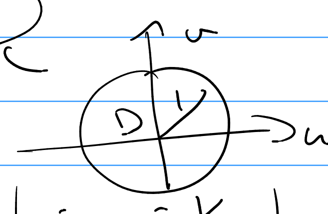

$$\iint_S (\vec{F}\cdot\vec{n})\,dS =$$

$S$: $x = u$ or $\vec{r}(u,v) = \langle u, v, u^2+v^2\rangle$

$y = v$ $\quad \vec{r}_u = \langle 1, 0, 2u\rangle$

$z = u^2+v^2$ $\quad \vec{r}_v = \langle 0, 1, 2v\rangle$

use $$\iint_S (\vec{F}\cdot\vec{n})\,dS = \iint_D \left(\vec{F}\cdot(\vec{r}_u\times\vec{r}_v)\right) dA$$

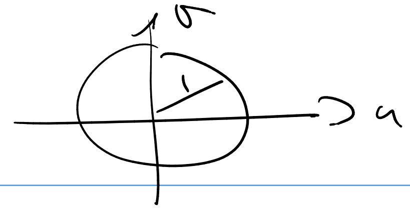



$\vec{F} = \langle x, y, 1\rangle$

$= \langle u, v, 1\rangle$

(sw) $$\vec{r}_u\times\vec{r}_v = \begin{vmatrix} i & j & k \\ 1 & 0 & 2u \\ 0 & 1 & 2v \end{vmatrix} = \langle -2u, -2v, 1\rangle$$

So $\iint_S (\vec{F}\cdot\vec{n})\,dS = \iint_D (1-2u^2-2v^2)\,dA$




in polar $= \int_0^{2\pi}\int_0^1 (1-2r^2)\,r\,dr\,d\theta = \frac{e\pi}{2}$

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## Stoke's thm


$S$ is an oriented piecewise smooth surface
$\partial S$ is a closed piecewise smooth with pos. orientation

**then** $\iint_S (\text{curl}(\vec{F}))\cdot d\vec{S} = \oint_{\partial S} \vec{F}\cdot d\vec{r}$

$\nabla \times \vec{F}$