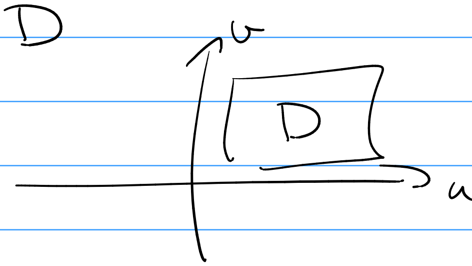
Math 344

S : $\vec{r}(u,v) = \langle x(u,v), y(u,v), z(u,v) \rangle$

$u,v \in D$



$\iint_S (f)\, dS$ Surface integral

$\iint_S \vec{F} \cdot d\vec{S} = \iint_S (\vec{F} \cdot \vec{n})\, dS$ surface integral of a vector field

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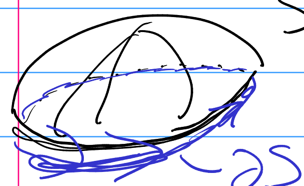
$\boxed{\text{Stokes' thm}}$ $\iint_S (\nabla \times \vec{F}) \cdot d\vec{S} = \oint_{\partial S} \vec{F} \cdot d\vec{r}$

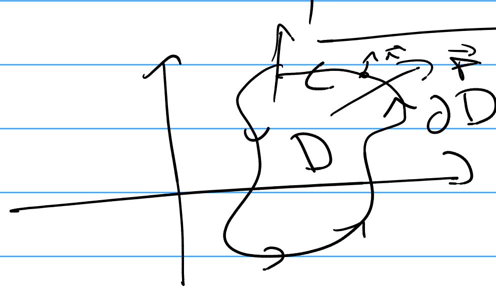
before: $\iint_S (\nabla \times \vec{F}) \cdot d\vec{S} = \iint_S (\nabla \times \vec{F}) \cdot \vec{n}\, dS$

ex S is a 'dome' $= \iint_D (\nabla \times \vec{F}) \cdot (\vec{r}_u \times \vec{r}_v)\, dA$



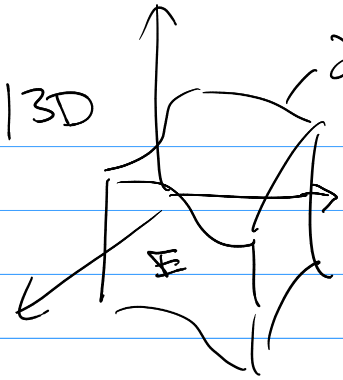
$\boxed{\text{Divergence thm}}$

1D/2D $\boxed{\text{Green's thm}}$



$\oint_{\partial D} (\vec{F} \cdot \vec{n})\, ds = \iint_D \operatorname{div}(\vec{F})\, dA$

2D|3D



$\vec F = \langle P, Q, R \rangle$

$$\iint_{\partial E} \vec F \cdot d\vec S = \iint_{\partial E} (\vec F \cdot \vec n)\, dS = \iiint_E \mathrm{div}(\vec F)\, dV$$

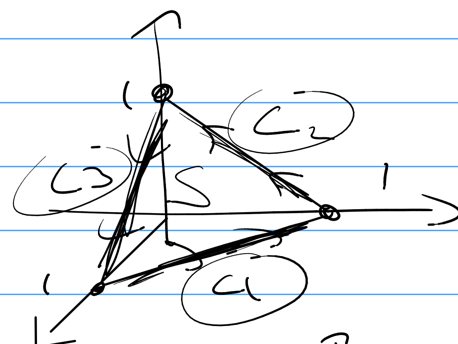
$(P_x + Q_y + R_z)$

$\boxed{\text{Stokes thm}}$

$$\iint_S (\mathrm{curl}(\vec F)) \cdot d\vec S = \oint_{\partial S} \vec F \cdot d\vec r$$

ex $\oint \langle x+y^2, y+z^2, z+x^2 \rangle \cdot d\vec r$

$C \to \vec r(t)$



$\vec F = \langle x+y^2, y+z^2, z+x^2 \rangle$

$C = C_1 \cup C_2 \cup C_3$

w/o stokes thm

$$\oint_C \vec F \cdot d\vec r = \int_\alpha^\beta (\vec F(\vec r) \cdot \vec r')\, dt$$

$\vec r = \langle x(t), y(t), z(t) \rangle$

$t \in [\alpha, \beta]$

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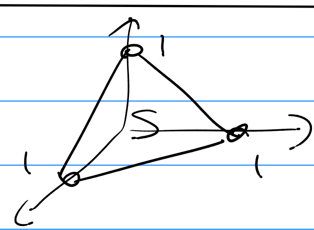
with Stokes need $\boxed{S}$

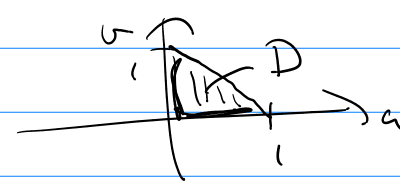


$x + y + z = 1 : S$

S: $x = u$
$y = v$
$z = 1 - u - v$

$\vec r(u,v) = \langle u, v, 1-u-v \rangle$

$$\iint_S (\mathrm{curl}(\vec F)) \cdot d\vec S =$$



$$\vec{F} = \langle \overset{P}{x+y^2}, \overset{Q}{y+z^2}, \overset{R}{z+x^2} \rangle$$

$$\text{curl}(\vec{F}) = \nabla \times \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = \langle \quad , \quad , \quad \rangle$$

Finish!