Math 344

Monday: Review (Fall)

Wed: Exam 4

Friday: no classes → have a review for final.

Wed May 9th @ 7am
Final Exam

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Exam 1
Exam 2
Exam 3
Exam 4

16 probs

Exam 4 — Vector Fields..

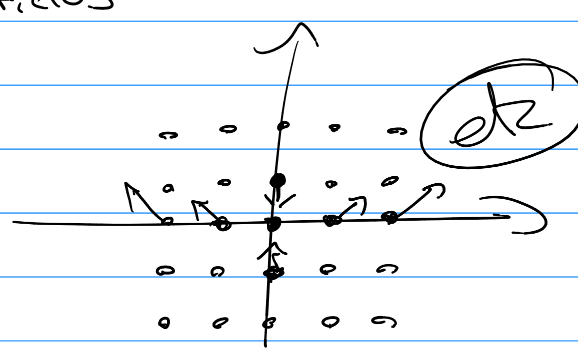
16.1 Vector field $\vec{F} = \langle P, Q \rangle$ 2D

$\vec{F} = \langle P, Q, R \rangle$ 3D

1 prob

Draw 2D Vector Fields

ex $F = \langle x, x-y \rangle$



16.2 Line Integrals

$C: \vec{r}(t) = \langle x(t), y(t), z(t) \rangle$

2 probs

① use $\int_C f\, ds = \int_\alpha^\beta f(\vec{r})\, |\vec{r}'|\, dt$

② use $\int_C \vec{F} \cdot d\vec{r} = \int_C \vec{F} \cdot \vec{T}\, ds = \int_\alpha^\beta \left(\vec{F}(\vec{r}) \cdot \vec{r}'\right) dt$

ex $f = xy$ $\quad \vec{r}(t) = \langle t, t^2, 1 \rangle \quad t \in [0,1]$

$\vec{r}' = \langle 1, 2t, 0 \rangle$

$$\int_C f\, ds = \int_0^1 t^3 \sqrt{1 + 4t^2}\, dt$$

16.3 Fund. thm

$\int_C \vec{F} \cdot d\vec{r}$

$\hookrightarrow$ if $\vec{F} = \nabla f$, then $\int_C \vec{F} \cdot d\vec{r} = f(\vec{r}(\beta)) - f(\vec{r}(\alpha))$

2 probs ① is $\vec{F}$ conservative? $\quad (F = \langle P, Q \rangle)$

2D $\quad$ is $\vec{F} = \nabla f$? check $P_y = Q_x$

yes?

② if $\vec{F}$ is conservative, then find f.

ex $\vec{F} = \langle \underbrace{2x + y^2}_{P}, \underbrace{2yx + \sin(y)}_{Q} \rangle$

is $\vec{F}$ conservative?

$P_y = 2y$

$Q_x = 2y$ $\rightarrow$ yes!

So $\vec{F} = \nabla f = \langle f_x, f_y \rangle$

$f_x = 2x + y^2$ $\rightarrow$ $\boxed{f = x^2 + xy^2 + C(y)}$

$f_y = 2xy + \sin(y)$

compare

b/c $f = x^2 + xy^2 + C(y)$

$\rightarrow f_y = 2xy + C'(y)$

So $C'(y) = \sin y \Rightarrow C(y) = -\cos(y) + C$

$$\boxed{f(x,y) = x^2 + xy^2 - \cos(y) + C}$$

problem #2 use Fund. th^m.

ex $\vec{F} = \langle 2x + y^2, 2xy + \sin y \rangle$ $C: \langle e^t \sin t^3, \cos t^{1/3} \rangle$

$t \in [0,1]$

$t = 0$ $(0,1)$

$t = 1$ $(e \sin(1), \cos(1))$

use f

$$\int_C \vec{F} \cdot d\vec{r} = f(e\sin(1), \cos(1)) - f(0,1)$$

16.4 Green's th^m 2 probs.

$$\oint_C \vec{F} \cdot d\vec{r} = \iint_D (Q_x - P_y)\, dA$$

$F = \langle P, Q \rangle$

① $\displaystyle\oint_C \vec{F}\cdot d\vec{r} = \boxed{\iint_D (Q_x - P_y)\,dA}$

do this side

② $\displaystyle\iint_D (Q_x - P_y)\,dA = \boxed{\oint_C \vec{F}\cdot d\vec{r}}$

do this side

Note: this will be $\displaystyle\iint_D (1)\,dA$

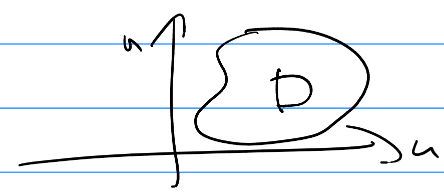
16.5 ① $\text{curl}(\vec{F}) = \nabla \times \vec{F}$

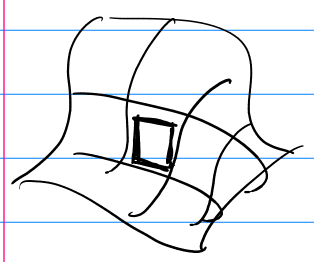
2 probs

② $\text{div}(\vec{F}) = \nabla \cdot \vec{F}$

16.6 $S: \vec{r}(u,v) = \langle x(u,v), y(u,v), z(u,v) \rangle$

$u, v \in D$





S

1 prob

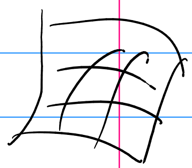
area of S $= A(S) = \displaystyle\iint_D |\vec{r}_u \times \vec{r}_v|\,dA$

16.7 Surface Integrals

1 prob

$$\iint_S f\,dS = \iint_D f(\vec{r})\,|\vec{r}_u \times \vec{r}_v|\,dA$$

ar



$$\iint_S \vec{F} \cdot d\vec{S} = \iint_S (\vec{F} \cdot \vec{n}) \, dS = \iint_D \left(\vec{F}(\vec{r}) \cdot (\vec{r}_u \times \vec{r}_v) \right) dA$$

$$\iint_S \vec{F} \cdot d\vec{r} = \iiint_E \operatorname{div} \vec{F} \, dV$$