Math 344

Final Exam

16 probs @ 10 pts each

140 pts = 100%

give in one of my exams

Exam 1 → "take" 4 problems ex $\int \langle x\sqrt{x^2+1}, \sec^2(x), \frac{1}{x^2+x} \rangle dx$

"take":
1. same problem.
2. small changes → $\int \langle x(x^2+2)^{1/3}, \sec^2(x), \frac{3}{x^2+x} \rangle dx$
3. same concepts → $\int \langle t^2\sin(t^3), \cosh(t), \frac{1}{(t+1)(t-1)} \rangle dt$

Exam 2 → take 4 probs

Exam 3 → take 4 probs

Exam 4 → take 4 probs

Math 344 ... Final Exam Review

Exam 1 Problems ... 4 problems will be used

1) Find the equation of the plane passing through the points (1,0,1), (-1,1,3), and (4,2,2).

(a,0,0) (0,b,0) (0,0,c)

2) Find an equation of a plane with x-intercept a, y-intercept b, and z-intercept c.

3) Classify, write equation in standard form, and sketch: $x^2+y^2-2x-6y-z+10=0$

Conic sections

4) Classify, write equation in standard from, and sketch: $4x^2+y^2+z^2-24x-8y+4z+55=0$

5) Find a vector function that represents the curve of intersection of the cylinder $x^2+y^2=4$ and the surface $z=xy$.

$\boxed{f:\mathbb{R}\to \text{Vectors}}$ 2D, 3D

6) Find the limit ...

$$\lim_{t\to\infty}\left\langle te^{-t}, \frac{t^2+t}{3t^2-1}, t\sin\left(\frac{1}{t}\right)\right\rangle$$

7) If $r(t)=\langle e^{2t}, e^{-2t}, te^{2t}\rangle$ find $\boldsymbol{T}(t)$ and $\boldsymbol{r}'(t)\cdot\boldsymbol{r}''(t)$.

8) Evaluate the integral ...

$$\int\left\langle te^{3t+1}, \frac{1+t}{1+t^2}, \frac{\cosh(t)}{\cosh^2(t)-1}\right\rangle dt$$

9) Find the length of the curve: $\boldsymbol{r}(t)=\langle t^2, 9t, 4t^{3/2}\rangle$, $1\leqslant t\leqslant 4$.

10) Use Section 13.3 Theorem 10 to find the curvature: $\boldsymbol{r}(t)=\langle t, t^2, e^t\rangle$.

11) Show that a projectile reaches 75% of its maximum height in half the time needed to reach its maximum height.

12) A ball with mass 0.6 kg is thrown southward into the air with a speed of 25 m/s at an angle of 40^o to the ground. A west wind applies a steady force of 3 N to the ball in an easterly direction. Where does the ball land and with what speed?

$f(x,y)$ or $f(x,y,z)$

$f : \mathbb{R}^n \to \mathbb{R}$

$n=2$ or $n=3$

EXAM 2 PROBLEMS ... 4 PROBLEMS WILL BE USED

~~1)~~ Show that the limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$$

~~2)~~ Find the first partials of the function $f(x,y,z) = x^3yz^2 + \sin(xy)$

3) Find the second partials of the function $f(x,y) = x^3 - \frac{1}{y} + \sin(x)\cos(y)$

~~4)~~ Find the linearization $L(x,y)$ of the function $f(x,y) = x\sin(x+y)$ at (-1,1).

?

5) Use differentials to estimate the maximum error in calculating the surface area of a right circular cylinder if you measure its height to be 10 ft ± 0.5 in and it's diameter to be 6 ft ± 1 in.

?

6) If $f(x,y) = e^x \sin(y)$, where $x(s,t) = s^2 + t^2$ and $y(s,t) = st^2$. Find the first partials f_s and f_t.

by chain rule

?

7) You find the temperature function for a heated metal plate at a given point (x,y) to be $T(x,y) = 30 + 2\sin(x^2+y^2)$ where T is measured in Celsius and x, y in cm. Find the maximum rate of increase at the point (1,2) and its direction. (gradient)

~~8)~~ Find and classify the critical points of the function $f(x,y) = x^4 + y^4 - 4xy$

~~9)~~ Find the absolute extrema of the function $f(x,y) = x^2 - 2xy + 2y$ on the rectangle $0 \leqslant x \leqslant 2$, $0 \leqslant y \leqslant 3$.

~~10)~~ Use Lagrange multipliers to find the extreme values of the function $f(x,y) = xy$ subject to the constraint $4x^2 + y^2 = 8$.

?

11) Use Lagrange multipliers to setup the system of equations that would find the extreme values of $f(x,y,z) = yz + xy$ subject to the two constraints $xy = 1$ and $y^2 + z^2 = 1$. Just setup the system ... DO NOT SOLVE the system.

$f(x,y)$ or $f(x,y,z)$ $\quad$ $f: \mathbb{R}^n \to \mathbb{R}$ $\quad$ $n=2,\ n=3$

Exam 3 Problems ... 4 problems will be used

1) Calculate the double integral

$$\iint_R \left(x+\frac{y}{x^2}\right)dA, \qquad R=\{(x,y)|0\leqslant x\leqslant 1, 0\leqslant y\leqslant 2\}$$

2) Setup the iterated integral for the given double integral using the order $dx\,dy$. DO NOT SOLVE the integral.

$$\iint_R (x e^{yx})dA, \qquad R \text{ is the region bounded by } y=x+2 \quad \text{and} \quad y=x^2$$

3) Use polar coordinates to find the volume under $z=x^2+y^2+x$ above the xy-region bounded by a circle of radius 1 centered at the origin, to the left of $y=x$, and above the x-axis.

4) Setup the three iterated integrals to find the center of mass of a triangular lamina with vertices (0,0), (1,0), and (0,2) if the density function is $\rho(x,y)=1+x+y$. DO NOT SOLVE the integrals.

5) Setup the iterated integral to find the surface area of the part of the paraboloid $x=x^2+y^2$ that lies under the plane $z=4$. Then convert that iterated integral into polar coordinates. DO NOT SOLVE the iterated integrals.

6) Evalute the integral.

$$\iiint_R (6xy^2+3z^2)dA, \qquad R=\{(x,y)|0\leqslant x\leqslant 1, 0\leqslant y\leqslant 2, -1\leqslant z\leqslant 2\}$$

7) Setup the triple iterated integral to evaluate ... ?

$$\iiint_R z\,dA, \qquad R \text{ is the solid tetrahedron bounded by } x=0,\ y=0,\ z=0, \text{ and } z=1-x-y$$

8) Use cylindrical coordinates to evaluate $\iiint_R \sqrt{x^2+y^2}\,dV$, where R is the region that lies inside the cylinder $x^2+y^2=1$ and between the planes $z=0$ and $z=2$.

9) Use spherical coordinates to setup the iterated integral for $\iiint_R f(x,y,z)dV$, where R is the upper half-sphere centered at the origin with radius 2. DO NOT SOLVE the iterated integral.

10) Find the Jacobian of the transformation $x=s\cos(t)$, $y=s\sin(t)$.

11) Setup the iterated integral to evaluate $\iiint_R (x^2 + y^2 - z)\,dV$, where R is the solid enclosed by ...

$x^2 + y^2 - z = 0,\ x^2 + y^2 - z = 1,$

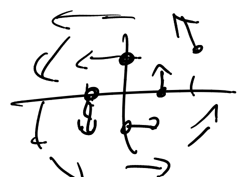
$x^2 - y^2 - z = 0,\ x^2 - y^2 - z = 2,$

$y^2 - x^2 - z = 0,\ y^2 - x^2 - z = 3,$

using the transformation $u = x^2 + y^2 - z$, $v = x^2 - y^2 - z$, and $w = y^2 - x^2 - z$. Leave your Jacobian in determinant form. DO NOT SOLVE the iterated integral.

Exam 4 Problems ... 4 problems will be used

1) Sketch a 2D Vector Field for $\boldsymbol{F}(x, y) = \langle -y, x \rangle$



? 0

2) Evaluate the Line Integral $\int_C (1 + x^2 y)\,ds$, where C is given by $\boldsymbol{r}(t) = \langle \sin(t), \cos(t) \rangle$ for $t = 0$ to $t = \pi$.

$\int_C f\,ds = \int_0^\pi (1 + \sin^2 t \cos t)\,dt$ $\quad \bar{x} \quad \bar{y}$

$\vec{r}\,' = \langle \cos t, -\sin t \rangle$

? 6

3) Evaluate the Line Integral of a Vector Field $\int_C \boldsymbol{F} \cdot d\boldsymbol{r}$ where $\boldsymbol{F}(x, y, z) = \langle xy, yz, zx \rangle$ and C is the space curve $r(t) = \langle t, t^2, t^3 \rangle$ from $t = 0$ to $t = 1$.

$\int_C \vec{F} \cdot d\vec{r} = \int_0^1 \langle t^3, t^5, t^4 \rangle \cdot \langle 1, 2t, 3t^2 \rangle\,dt$

4) Is $\boldsymbol{F}(x, y) = \langle xy + y^2, xy + x^2 \rangle$ a conservative vector field? And why or why not?

5) Evaluate $\int_C \boldsymbol{F} \cdot d\boldsymbol{r}$ where $F(x, y) = \langle x^2 y^3, x^3 y^2 \rangle$ and C is $r(t) = \langle t^3 - 2t, t^3 + 2t \rangle$ from $t = 0$ to $t = 1$ by using the Fundamental Theorem for Line Integrals.

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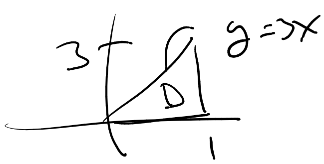
6) Use Green's Theorem to setup the Double Integral to evaluate the Line Integral ...



$$\int_C \langle x^2 y^3, \sqrt{y + x^2} \rangle \cdot d\boldsymbol{r}$$

where C is the closed curve from the points (0,0) to (1,0) to (1,3) and then back to (0,0).

DO NOT SOLVE the Double Integral.

7) Use Green's Theorem to setup the Line Integral to evaluate the Double Integral ...

$$\iint_D (1)\,dA$$ See textbook.

where D is the ellipse $x^2 / a^2 + y^2 / b^2 = 1$ with parametric equations $x = a\cos(t)$ and $y = b\sin(t)$ with $t = 0$ to $t = 2\pi$. DO NOT SOLVE the Line Integral.

? 0

8) Calculate the curl of $\boldsymbol{F}(x,y,z) = \langle xz, xyz, -y^2\rangle$

$\nabla \times \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xz & xyz & -y^2 \end{vmatrix} =$

? 0

9) Calculate the divergence $\boldsymbol{F}(x,y,z) = \langle xz - \tan(y), xyz + \cos(x), -y^2 + \tanh(xy)\rangle$

$4 = x^2 + y^2$

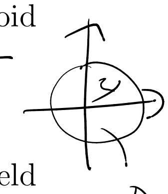
$\nabla \cdot \vec{F} =$

~~10)~~ ~~Setup~~ the Double Integral in polar coordinates to find the area of the part of the paraboloid $z = \cancel{x^2} + y^2$ under the plane $z = 4$. DO NOT SOLVE the Double Integral.

$A(S) = \iint_S |\vec{r}_u \times \vec{r}_v| \, dS$

$x = u \quad y = v$

$z = u^2 + v^2$



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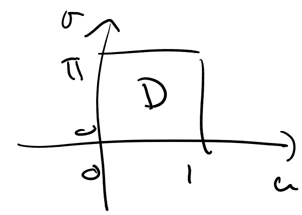
11) Setup the Double Integral to evaluate the Surface Integral $\iint_S \boldsymbol{F} \cdot d\,\boldsymbol{S}$ for the vector field $\boldsymbol{F}(x, y, z) = \langle z, y, z\rangle$ and S is the helicoid $r(u, v) = \langle u\cos(v), u\sin(v), v\rangle$, $0 \leqslant u \leqslant 1$, $0 \leqslant v \leqslant \pi$. DO NOT SOLVE the Double Integral.

D

$\iint_S \vec{F} \cdot d\vec{S} = \iint_D \vec{F}(\vec{r}) \cdot (\vec{r}_u \times \vec{r}_v) \, dA$



or $\iint_S f \, dS = \iint f(\vec{r}) \, |\vec{r}_u \times \vec{r}_v| \, dA$