

Math 144

Q's! $\frac{d}{dx} \left[\frac{x^2 + 4x + 5}{x^{1/2}} \right] = ?$

$$\frac{a}{c} + \frac{b}{c} \stackrel{R}{=} \frac{a+b}{c} \quad \text{at} \quad \frac{a+b}{c} = c^{-1}(a+b)$$

$$\frac{d}{dx} \left[\frac{x}{x^{1/2}} + \frac{4x}{x^{1/2}} + \frac{5}{x^{1/2}} \right] = \frac{d}{dx} \left[x^{1/2} + 4x^{1/2} + 5x^{-1/2} \right]$$

$$= \text{etc}$$

Deriv Rules: $\boxed{\text{1st}}$ limit definition $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

$$\frac{d}{dx} [C] = 0$$

$$\frac{d}{dx} [C f(x)] = C f'(x)$$

$$\frac{d}{dx} [x] = 1$$

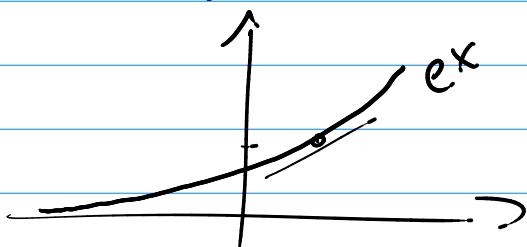
$$\frac{d}{dx} [f(x) \pm g(x)] = f'(x) \pm g'(x)$$

$$\frac{d}{dx} [x^n] = n x^{n-1}$$

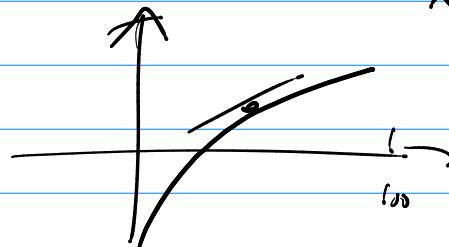
New Rules: exponential functions , logarithmic functions

Natural number $e = 2.71828 \dots$

$$\frac{d}{dx} [e^x] = e^x$$



$$\frac{d}{dx} [\ln(x)] = \frac{1}{x}, \quad x > 0$$



other basis:

$$b^x$$

$$\log_b x$$

$$\frac{d}{dx} [b^x] = (\ln b) b^x$$

$$\frac{d}{dx} [\log_b x] = \left(\frac{1}{\ln b}\right) \frac{1}{x}$$

(Ex) $\frac{d}{dx} [2x - x^{1/2} + e^x - 3 \ln x]$

$$\begin{aligned} &= (2x) - (x^{1/2})' + [e^x]' - [3 \ln x]' \\ &= 4x - \frac{1}{2}x^{-1/2} + (x - 3(\frac{1}{x})) \end{aligned}$$

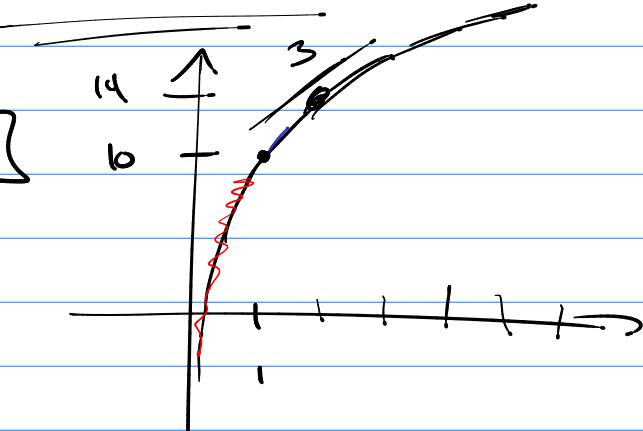
Why? (Ex) Number of words typed is..

$$N(t) = 10 + 6 \ln(t) \quad t \geq 1$$

$$N'(t) = \frac{d}{dt} [10 + 6 \ln(t)]$$

$$= 0 + 6(\frac{1}{t})$$

$$N'(t) = \frac{6}{t}$$



Q t = 10

$$N(10) = 10 + 6 \ln(10) \approx 23.815\dots$$

$$N'(10) = \frac{6}{10} = \frac{3}{5} = 0.6$$

Q t = 2

$$N(2) = 10 + 6 \ln(2) \approx 14.1\dots$$

$$N'(2) = \frac{6}{2} = 3$$

Q t = 100

$$N(100) = 10 + 6 \ln(100) \approx 37.6$$

$$N'(100) = \frac{6}{100} = 0.06$$

So far

Derivative Sheet

$$\frac{d}{dx}[c] = 0 \quad \text{make 5 graphs}$$

$$\frac{d}{dx}[x] = 1$$

$$\frac{d}{dx}[x^n] = nx^{n-1} \quad \text{make 5 examples}$$

$$\frac{d}{dx}[c f(x)] = c f'(x) \quad \text{make 5 examples}$$

$$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x) \quad \text{make 5 examples}$$

$$\frac{d}{dx}[e^x] = e^x$$

$$\frac{d}{dx}[b^x] = (\ln b) b^x$$

$$\frac{d}{dx}[\ln x] = \frac{1}{x}$$

$$\frac{d}{dx}[\log_b x] = \frac{1}{\ln b} \frac{1}{x}$$

what about?

$$(3x + x^2)(e^x - \ln x)$$

defn:

$$(x+1)(x^2-x+1) = x^3 - x^2 + x + x^2 - x + 1 \\ = x^3 + 1$$

type: $\frac{d}{dx}[f(x)g(x)]$

(ex) $\frac{d}{dx}[x e^x] = ?$

No rule? use limit defn.tra...

$$\frac{d}{dx}[f(x)g(x)] = \lim_{h \rightarrow 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

word hence been use f, g vs

$$\frac{f(x+h) - f(x)}{h}$$

$$\begin{aligned}
 &= \lim_{h \rightarrow 0} \frac{(f(x+h)g(x+h)) - (f(x)g(x+h)) + (f(x)g(x+h)) - (f(x)g(x))}{h} \\
 &= \lim_{h \rightarrow 0} \left[\frac{f(x+h) - f(x)}{h} g(x+h) \right] + f(x) \left[\frac{g(x+h) - g(x)}{h} \right] \\
 &= f'(x)g(x) + f(x)g'(x)
 \end{aligned}$$

So Product rule $\frac{d}{dx} \{f(x)g(x)\} = f'(x)g(x) + f(x)g'(x)$

Example: $\frac{d}{dx} \{x + e^x\} = [x]' + [e^x]' = \boxed{1 + e^x}$

(15) $\frac{d}{dx} \{x e^x\} = [x]' e^x + x [e^x]' = (1) e^x + x e^x = \boxed{e^x + x e^x}$

Ex $\frac{d}{dx} \{x^{1/2} \ln x\}$
 $= (x^{1/2})'(\ln x) + (x^{1/2})(\ln x)'$
 $= (\frac{1}{2}x^{-1/2})(\ln x) + (x^{1/2})\left(\frac{1}{x}\right)$

(15) $\frac{d}{dx} \{x^{1/2} + \ln x\} = \boxed{\frac{1}{2}x^{-1/2} + \frac{1}{x}}$

$$\begin{aligned}\frac{d}{dx} [3x^2 \ln x] &= [3x^2]'[\ln x] + [3x^2][\ln x]' \\ &= \underline{\underline{6x \ln x + 3x^2(\frac{1}{x})}} \\ &= \underline{\underline{6x \ln x + 3x}}\end{aligned}$$

$$\begin{aligned}\frac{d}{dx} [(x^2+x-1)(x^{1/3}-2x+4)] &= \\ \text{Product rule} \rightarrow &= [x^2+x-1] \overset{'}{[x^{1/3}-2x+4]} + [x^2+x-1] \circled{[x^{1/3}-2x+4]}' \\ \rightarrow &= [2x+1] \circled{[x^{1/3}-2x+4]} + [x^2+x-1] \underline{\underline{[\frac{1}{3}x^{-2/3}-2]}} \\ &= 2x^{4/3} - (x^2+8x+x^{1/3}-2x+4) \\ &\quad + \frac{1}{3}x^{-2/3} - 2x^2 + \frac{1}{3}x^{1/3} - 2x - \frac{1}{3}x^{-2/3} + 2 \\ &= \underline{\underline{e^{2x-2}}}\end{aligned}$$

$$\frac{d}{dx} \left\{ \frac{f(x)}{g(x)} \right\} \underset{\text{ex}}{=} \frac{d}{dx} = \left\{ \frac{x}{e^x} \right\}$$

$$\text{quotient rule} \quad \frac{d}{dx} \left\{ \frac{f(x)}{g(x)} \right\} = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

$$\textcircled{1} \quad \textcircled{2} \quad \frac{d}{dx} \{x + e^x\} = [x]' + [e^x]' = \boxed{1 + e^x}$$

$$\begin{aligned}\textcircled{3} \quad \frac{d}{dx} \{x e^x\} &= [x]'[e^x] + [x][e^x]' \\ &= \frac{(1)e^x + x e^x}{e^x + x e^x}\end{aligned}$$

$$\textcircled{#3} \quad \frac{d}{dx} \left\{ \frac{x}{e^x} \right\} = \frac{x e^x - [x] e^x}{[e^x]^2}$$

$$= \frac{(1)e^x - x e^x}{e^{2x}} = \frac{e^x - x e^x}{e^{2x}}$$

$$= \cancel{\left(\frac{e^x}{e^{2x}} \right)} \left[1 - x \right] = \frac{1-x}{e^{-x} e^{2x}} = \boxed{\frac{1-x}{e^x}}$$

$$\begin{array}{c} x^2 x^3 = x^5 \\ \hline \frac{e^x}{e^y} = e^{x-y} \\ \hline \frac{e^x}{e^y} = \frac{1}{e^{y-x}} \end{array}$$

$$\frac{d}{dx} \left\{ f(g(x)) \right\} \quad \textcircled{ex} \quad \frac{d}{dx} \left\{ \ln(x^2 + 2x) \right\}$$

Chain rule

$$\frac{d}{dx} \left\{ f(g(x)) \right\} = f'(g(x)) \cdot g'(x)$$

Generalized Power rule

$$\frac{d}{dx} \left\{ (f(x))^n \right\} = n (f(x))^{n-1} \cdot f'(x)$$

$$\textcircled{ex} \quad \frac{d}{dx} \left\{ (2x+1)^{\frac{1}{2}} \right\} = \frac{1}{2} (2x+1)^{-\frac{1}{2}} \frac{d}{dx} [2x+1]$$

$$= \frac{1}{2} (2x+1)^{-\frac{1}{2}} [2]$$

$$\frac{d}{dx} [\ln(x^2+2x)] = \frac{1}{x^2+2x} \cdot \frac{d}{dx}[x^2+2x]$$
$$= \boxed{\frac{1}{x^2+2x} [2x+2]}$$

~~or Simplify~~
$$= \frac{2x+2}{x^2+2x} = \boxed{\frac{2(x+1)}{x(x+2)}}$$