

Math 144

A formula for the relationship between weight and blood pressure in children is given by the formula below where $P(x)$ is measured in millimeters of mercury and x is measured in pounds. Use the formula to answer the questions.

$$P(x) = 16.6(1 + \ln x) \quad 10 \leq x \leq 100$$

What is the rate of change of blood pressure with respect to weight at the 20-pound weight level?

The rate of change at the 20-pound weight level is approximately $\boxed{}$ mm/pound.

(Do not round until the final answer. Then round to the nearest hundredth as needed.)

derivative

$$\begin{aligned} \frac{dP}{dx} &= \frac{d}{dx} [16.6 + 16.6 \ln x] \\ &= 0 + 16.6 \frac{1}{x} \end{aligned}$$

$$P'(20) = \frac{16.6}{20} = \frac{1.66}{2} = \boxed{0.83 \text{ mm/lb}}$$

#17 Graphing

geogebra.org

Rules for derivatives

$$[c]' = 0$$

$$[x^n]' = n x^{n-1}$$

$$-\frac{3}{4} - \frac{4}{4} = -\frac{7}{4}$$

$$[x]' = 1$$

$$[cx]' = c$$

$$[c f(x)]' = c f'(x)$$

$$[f(x) \pm g(x)]' = f'(x) \pm g'(x)$$

$$[e^x]' = e^x$$

$$[\ln x]' = \frac{1}{x}$$

$$[f(x)g(x)]' = f'(x)g(x) + f(x)g'(x)$$

$$\begin{aligned} (\ln x)^{-\frac{3}{4}} &= \frac{1}{2} (-\frac{3}{4}) x^{-\frac{5}{4}} \\ &= \boxed{-\frac{3}{8} x^{-\frac{5}{4}}} \end{aligned}$$

$$\left[\frac{f(x)}{g(x)} \right]' = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

$$[f(g(x))]' = f'(g(x))g'(x)$$

ex

$$\begin{aligned} & \left[\frac{2x+1}{x^2-1} + \ln(x^2+x+2) \right]' \\ &= \left[\frac{2x+1}{x^2-1} \right] + \left[\ln(x^2+x+2) \right]' \\ & \quad \text{quotient rule} \qquad \qquad \qquad \text{chain rule} \end{aligned}$$

$$\begin{aligned} &= \frac{[(2x+1)'(x^2-1) - (2x+1)(x^2-1)']}{(x^2-1)^2} + \frac{1}{x^2+x+2} [x^2+x+2]' \\ &= \frac{[(2)(x^2-1) - (2x+1)(2x)]}{(x^2-1)^2} + \frac{1}{x^2+x+2} (2x+1) \end{aligned}$$

$$= \frac{2x^2-2 - 4x^2-2x}{(x^2-1)^2} + \frac{2x+1}{x^2+x+2}$$

$$= \frac{-2x^2-2x-2}{(x^2-1)^2} + \frac{2x+1}{x^2+x+2} = \frac{-2(x^2+x+1)}{(x+1)^2(x-1)^2} + \frac{2x+1}{(x^2+x+2)}$$

$$= \frac{-2(x^2+x+1)(x^2+x+2) + (2x+1)(x+1)(x-1)^2}{(x+1)^2(x-1)^2(x^2+x+2)}$$

= ...

$$\begin{aligned} \frac{d}{dx} \left[\frac{3x+1}{x^2-1} \right] &= \frac{[3x+1]'(x^2-1) - [3x+1](x^2-1)'}{(x^2-1)^2} \\ &= \frac{[3(x^2-1) - (3x+1)(2x)]}{(x^2-1)^2} \\ &= \text{simplify using Algebra?} \end{aligned}$$

$$\frac{d}{dx} \left\{ (2x+x^3)^{\frac{4}{3}} \right\}$$

$$\begin{aligned} &= \frac{1}{3} (2x+x^3)^{-\frac{2}{3}} [2x+x^3]' \\ &= \boxed{\frac{1}{3} (2x+x^3)^{-\frac{2}{3}} (2+3x^2)} \end{aligned}$$

$$(a \cdot b)^n = a^n b^n$$

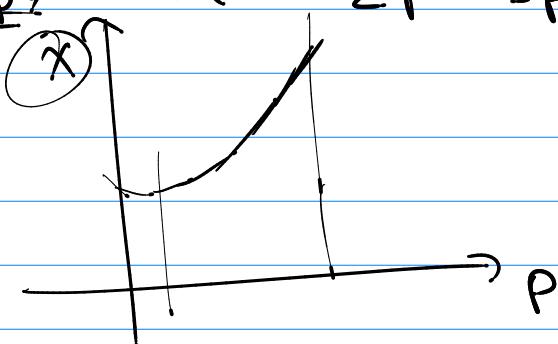
$$\cancel{(a+b)^n \neq a^n + b^n}$$

Application:

Price-Supply

fitness watches

$$X = \frac{1}{2} p^2 - 3p + 200$$



$$\begin{aligned} \frac{dX}{dp} &= [\frac{1}{2} p^2 - 3p + 200]' \\ &= p - 3 \frac{\text{units}}{\$} \end{aligned}$$

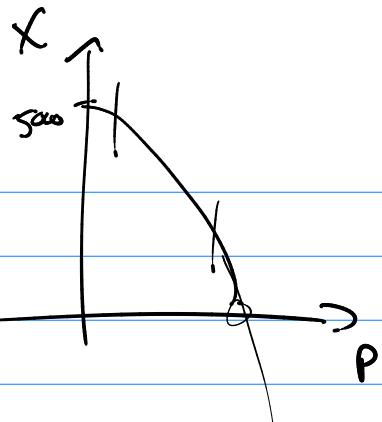
$$@ \$10 \quad X = \frac{1}{2}(10^2) - 3(10) + 200 = 220 \text{ units}$$

$$\frac{dX}{dp} = 10 - 3 = \frac{\text{units}}{\$}$$

Demand - Price

$$X = 5000 - 0.1 P^2$$

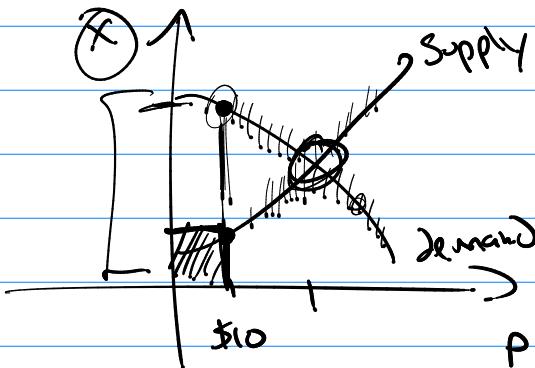
$$\frac{dx}{dp} = [5000 - 0.1 P^2]' \\ = -0.2 P$$



② 10\$ $X = 5000 - 0.1 (10)^2 = 4990 \text{ units}$

$$\frac{\Delta X}{\Delta P} = -0.2(10) = -2 \frac{\text{units}}{\$}$$

together?



Price changes with time! so $p(t)$
 Demand/Supply changes with time! so $x(t)$

Explicit vs Implicit functions

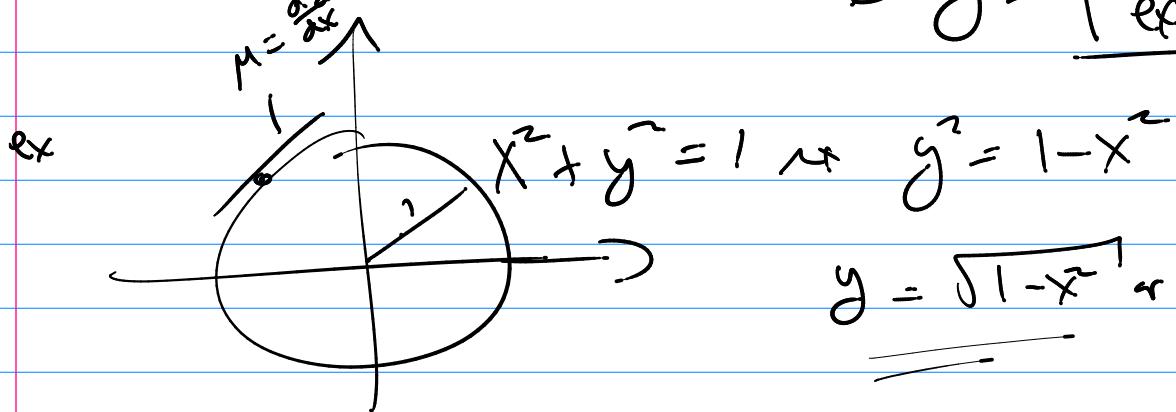
ex) $y = f(x)$

Explicit : means $y = [expression of x]$

ex) $y = 2x^2 + x - e^x$

$$y' = 4x + 1 - e^x$$

Implicit is $y = f(x)$ but y can not be made
 $\rightarrow y = \boxed{\text{expression}}$



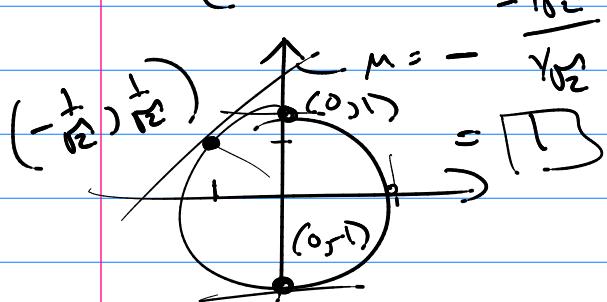
Can I find $y' = \frac{dy}{dx}$?

Implicit Derivatives: $x^2 + y^2 = 1$

The deriv.
of both sides
(remember $y = f(x)$)

$$\frac{d}{dx}[x^2 + y^2] = \frac{d}{dx}[1]$$

$$2x + (f(x))^2 = 0$$



$$(2x + 2yy') = 0$$

$$(2y)y' = -2x \quad \boxed{y' = -\frac{x}{y}}$$

$$y = f(x)$$

$$\frac{d}{dx}[y^3 + \ln y] = 3y^2 \cdot y' + \frac{1}{y} y'$$

back to demand

$$x = 5000 - 0.1 p^2$$

and I know $x(t)$, $p(t)$ are functions of t

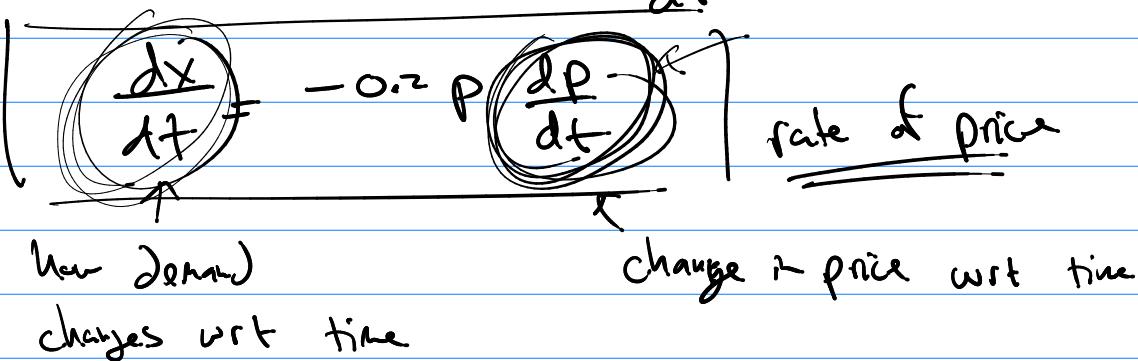
$$\frac{d}{dt}[x] = \frac{d}{dt}[5000 - 0.1 p^2]$$

(related)

rates
equation

rate of demand

$$1 \cdot \frac{dx}{dt} = -0.2 p \frac{dp}{dt}$$

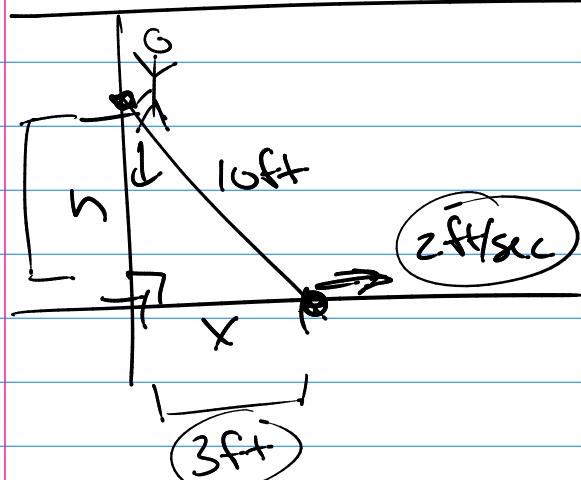


$$@ \$10 \text{ change price } @ \frac{\$0.50}{\text{wk}} = \frac{dp}{dt}$$

rate of demand (time)

$$\frac{dx}{dt} = -0.2 (10) (0.50)$$

$$= -\frac{1}{\text{wk}}$$



$$\bar{x}^2 + \bar{h}^2 = 10^2$$

$$\bar{x}^2 + \bar{h}^2 = 100$$

$$\frac{d}{dt}(\bar{x}^2 + \bar{h}^2) = \frac{d}{dt}(100)$$

$$2\bar{x} \frac{dx}{dt} + 2\bar{h} \frac{dh}{dt} = 0$$

$$x \frac{dx}{dt} + h \frac{dh}{dt} = 0$$

$$(3)(2) + \sqrt{11} \frac{dh}{dt} = 0$$

$$\frac{dh}{dt} = \frac{-6}{\sqrt{21}} \approx -0.63 \text{ ft/sec} \approx -7.5 \text{ in/sec}$$