

Math 144

Sample Exam

try #3, #5 & 3.1 - 3.4

try #1 & 3.6

try #1 & 3.7

do simplification:

$$\textcircled{1} \quad \frac{d}{dx} [x^3 - 2x^{1/3} + 7] = 3x^2 - \frac{2}{3}x^{-2/3}$$

$$\textcircled{2} \quad \frac{d}{dx} [ln x + 2e^x - x^{1/2}] = \frac{1}{x} + 2e^x - \frac{1}{2}x^{-1/2}$$

$$\textcircled{3} \quad \frac{d}{dx} [(2x^2 - x) \ln x] = (\underline{4x-1}) \underline{\ln x} + (2x^2 - x) \left(\frac{1}{x}\right)$$

$$\textcircled{4} \quad \frac{d}{dx} \left[\frac{x^{4/3} + 2x}{e^x - x^2} \right] = \frac{\left(\frac{4}{3}x^{-2/3} + 2\right)(e^x - x^2) - (x^{1/3} + 2x)(e^x - 2x)}{(e^x - x^2)^2}$$

$$\textcircled{5} \quad \frac{d}{dx} [(e^x + 2x^3)^{1/2}] = \frac{1}{2}(e^x + 2x^3)^{-1/2} (e^x + 6x^2)$$

35) $\frac{d}{dx} [3^2 - 2xy + x^3] = \frac{d}{dx} [x - 2y]$

$$6y \frac{dy}{dx} - [2y + 2x \frac{dy}{dx}] + 3x^2 = 1 - 2 \frac{dy}{dx} \quad \frac{dy}{dx} = ?$$

36) $x^2 + y^2 = x + y$

related rates $\frac{d}{dt}$

$$\frac{d}{dt} [x^2 + y^2] = \frac{d}{dt} [x + y]$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 1 \cdot \frac{dx}{dt} + 1 \cdot \frac{dy}{dt}$$

$$2(1)(2) + 2(3) \frac{dy}{dt} = 2 + \frac{dy}{dt} \quad 4 + 6 \frac{dy}{dt} = 2 + \frac{dy}{dt}$$

$$5 \frac{dy}{dt} = -2 \quad \underline{\underline{\frac{dy}{dt} = -\frac{2}{5}}}$$

$$\frac{dx}{dt} = 2 \\ x = 1 \\ y = 3$$

3.7

$$\textcircled{1} \quad \frac{f'(x)}{f(x)} = ? \quad \textcircled{2} \quad f(x) = (4x^3)(e^x + \ln x)$$

$$f'(x) = 12x^2(e^x + \ln x) + (4x^3)(e^x + \frac{1}{x})$$

$$\frac{f'(x)}{f(x)} = \frac{12x^2(e^x + \ln x) + 4x^3(e^x + \frac{1}{x})}{4x^3(e^x + \ln x)}$$

so just take $\underline{\underline{x}}$

Exam

13 probs @ 10pts each 120 pts = 100%

3.1 to 3.4 Derivatives (7 probs)

(a) $f(s) = \frac{d}{ds}[s^2 + 2s + 1]$
 (b) $\frac{d}{dx} \{ x^3 - x^{-\frac{1}{3}} \}$

Applications 6 probs

2 - Implicit Deriv. (3.5)

2 - Related Rates (3.6)

2 - Elasticity of Demand (3.7)

Why?

① - graphing

- finding special parts of a graph

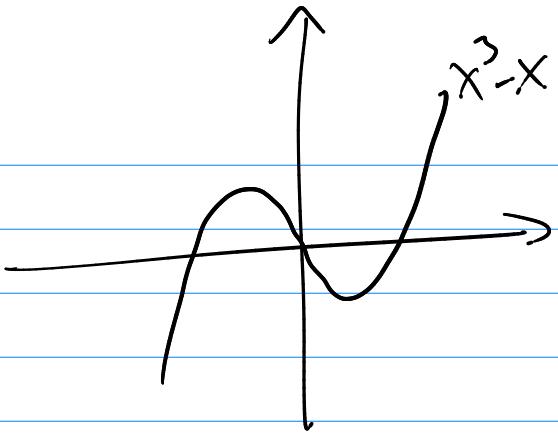
4.1/4.2

$$y = x^3 - x$$

Graph: college algebra

Plot Points

x	y
0	0
-1	0
1	0
2	6
-2	-6



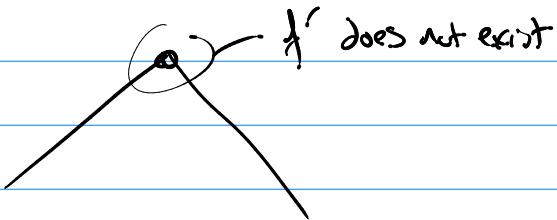
(calculus)

positive slope > 0

slope = 0

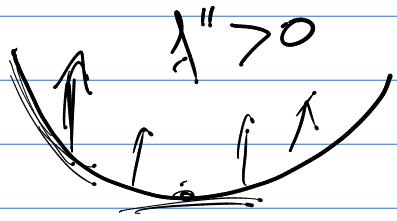
$$\frac{dy}{dx} = f' \quad (\text{slope})$$

slope < 0

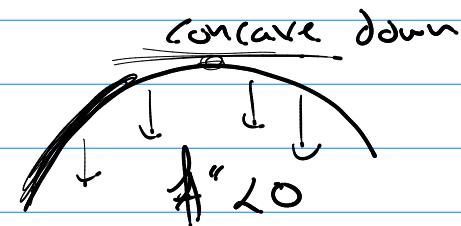


slope = 0

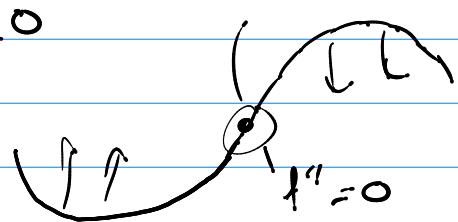
$$\frac{d}{dx}[f'] = \frac{d^2 f}{dx^2} = f'' \quad \text{second derivative.}$$



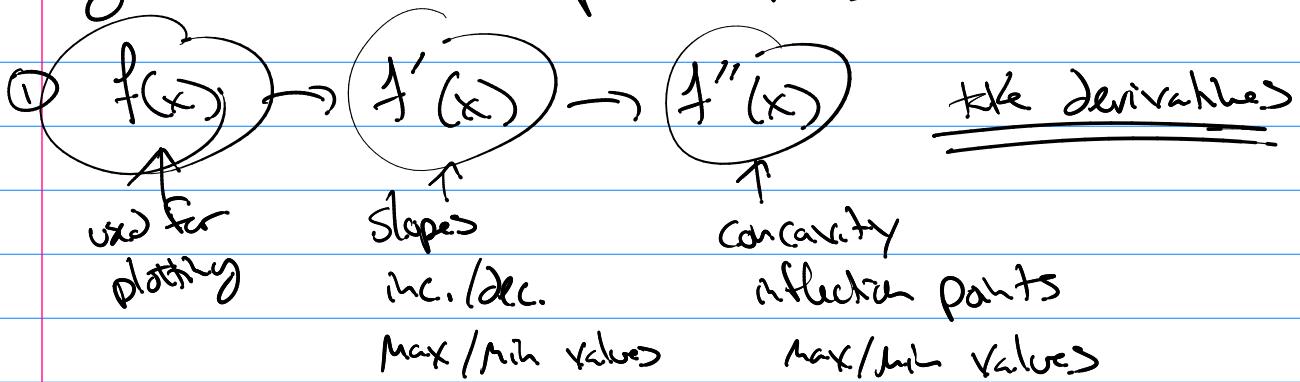
Concave up



inflection point

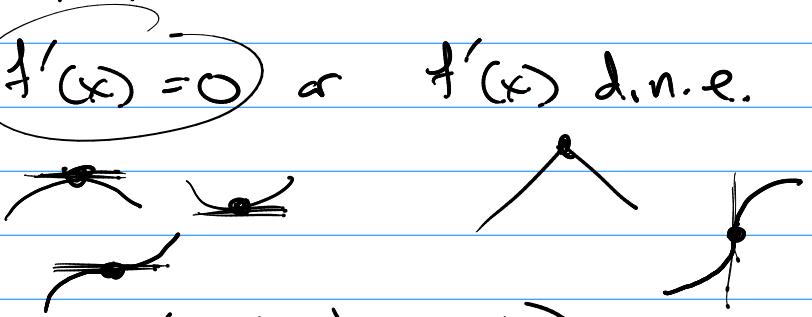


Using derivatives to plot $f(x)$



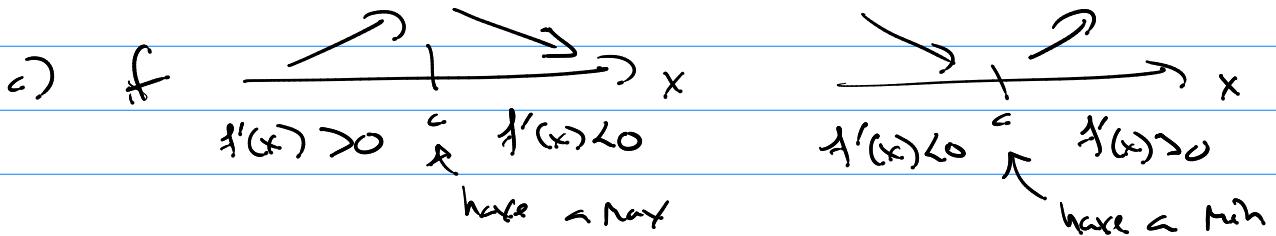
- ② use $f(x)$
- a) table of values
 - b) intercepts
 - c) asymptotes

- ③ use $f'(x)$
- a) $f'(x) = 0$ or $f'(x)$ d.n.e.



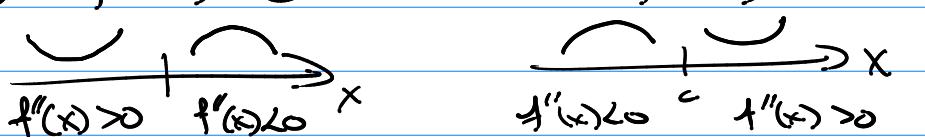
find the x -values (critical points)

- b) $f'(x) > 0$ or $f'(x) < 0$ regions
- inc. dec.



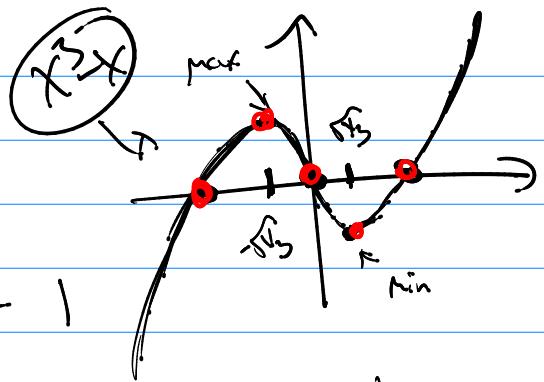
- d) plug our critical values back into table of values to find coords to plot

- ④ use f''
- a) $f''(x) = 0$ or $f''(x)$ dne
 - b) $f''(x) > 0$ or $f''(x) < 0$



c) put values into table of values to find card.

Plot $f(x) = x^3 - x$



① take derivatives

$$f(x) = x^3 - x$$

$$f'(x) = 3x^2 - 1$$

$$f''(x) = 6x$$

② use $f(x) = x^3 - x$

Intercept: $x=0$

(let $y=0$)

$$0 = x^3 - x$$

$$0 = x(x^2 - 1)$$

$$0 = x(x+1)(x-1)$$

$$x=0, x=-1, x=1$$

$y=0$

(let $x=0$)

$$y = 0^3 - 0 = 0$$

<u>X</u>	<u>$y = x^3 - x$</u>
0	0
1	0
-1	0
$\sqrt[3]{3}$	$(\sqrt[3]{3}) - (\sqrt[3]{3})^2$
$-\sqrt[3]{3}$	$-(\sqrt[3]{3}) + (\sqrt[3]{3})^2$

Max

asymptotes

Achse

$$\lim_{\substack{x \rightarrow 0 \\ x \rightarrow \infty}} x^3 - x = +\infty$$

$$\lim_{\substack{x \rightarrow 0 \\ x \rightarrow -\infty}} x^3 - x = -\infty$$

③ use $f'(x) = 3x^2 - 1$

$$\Leftrightarrow f'(x) = 0 \quad \leftarrow$$

$$3x^2 - 1 = 0$$

$$x^2 = \sqrt[3]{3}$$

$$x = \pm \sqrt[3]{3}$$

$f'(x)$ d.h.e never!

~~$f'(x) = 3x^2 - 1$~~

inc/dec
(sign table)

$$f'(x) = 2$$

$$f'(0) = -1$$

$$f'(\sqrt[3]{3}) = 2$$

$$f'(-\sqrt[3]{3}) = 2$$

c) $\max @ x = -\sqrt{y_3}$ $\min @ x = \sqrt{y_3}$

d) $x = -\sqrt{y_3}, \sqrt{y_3}$ to take

(a) $\text{or } f''(x) = 6x$

a) $f''(x) = 0$

$6x = 0$

$x = 0$

$f''(x)$ due 
 never

b)



$f''(-1) = -6$ $f''(1) = 6$ $f''(x) = 6x$

c) $x=0$ inf. point

put $x=0$ into take