Math 144

Q's Exam 4 (Note for Wed)

↳ review for final

① Final = 16 probs, 4 from Exam 1, 2, 3, 4

② Review Exams 1-4
- Any problems you don't want on final?
- Any problems you do want on final?

③ Exam 5 over ch 7 - take home only

④ Final (obviously) covers Exams 1 to 4 so...
→ if your final % is greater than your lowest % exam

(ex) Exam 1 = 70%
Exam 2 = 60%
Exam 3 = 72%
Exam 4 = ~~50%~~ 63%

Final: 63%

Exam 5: Covers

① Evaluate $F(x,y)$

$\frac{d}{dx}\left\{\frac{1}{3}x\right\} = \frac{1}{3}$

$\frac{\partial}{\partial x}\left\{\frac{x}{z}\right\} = \frac{1}{z}$

(ex) $F(x,y,z) = \boxed{x + y^2 z - \frac{x}{z}}$

$F(3,1,2) = 3 + 1^2 \cdot 2 - \frac{3}{2}$

② Partial Derivatives

(ex) $\frac{\partial F}{\partial x} = \frac{\partial}{\partial x}\left[x + y^2 z - \frac{x}{z}\right] = 1 + 0 - \frac{1}{z}$

$= \boxed{1 - \frac{1}{z}}$

③ Double integrals

$$\int_0^2 \left[\int_0^1 (x + yx^2)\,dx \right] dy$$

$$= \int_0^2 \left[\left[\tfrac{1}{2}x^2 + \tfrac{1}{3}yx^3 \right] \Big|_{x=0}^{x=1} \right] dy$$

$$= \int_0^2 \left[\left[\tfrac{1}{2} + \tfrac{1}{3}y \right] - [0] \right] dy$$

$$= \int_0^2 \left(\tfrac{1}{2} + \tfrac{1}{3}y \right) dy = \left[\tfrac{1}{2}y + \tfrac{1}{6}y^2 \right] \Big|_{y=0}^{y=2}$$

$$= \left[1 + \tfrac{2}{3} \right] - [0] = \boxed{\tfrac{5}{3}}$$

Exam 4

① a) $\int (x^3 - x^{1/2} + e^x - \tfrac{1}{x})\,dx$

$$= \tfrac{1}{4}x^4 - \tfrac{2}{3}x^{3/2} + e^x - \ln|x| + c$$

b) $\int (2x-1)(x^{1/2} + x^2)\,dx = \int (2x^{3/2} + 2x^3 - x^{1/2} - x^2)\,dx$

c) $\int \frac{(x+1)(x-1)}{x^2}\,dx = \int \frac{x^2-1}{x^2}\,dx$

$$= \int \left(\frac{x^2}{x^2} - \frac{1}{x^2} \right) dx = \int (1 - x^{-2})\,dx$$

Substitution

3c) $\int x\sqrt{2x-1}\,dx$

$u = 2x-1 \rightarrow x = \frac{u+1}{2}$

$du = 2dx$

$= \int \left(\frac{u+1}{2}\right)\sqrt{u}\,\frac{du}{2} = \frac{1}{4}\int (u+1)\,u^{1/2}du$

$= \frac{1}{4}\left[\int u^{3/2} + u^{1/2}\,du\right] = \frac{1}{4}\left[\frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2}\right] + C$

$= \frac{1}{10}(2x+1)^{5/2} + \frac{1}{6}(2x+1)^{3/2} + C$

Kahuna

$\int x^3 e^{x^2}dx = \frac{1}{2}x^2e^{x^2} - \int xe^{x^2}dx = \frac{1}{2}x^2e^{x^2} - \frac{1}{2}e^{x^2} + C$

$u = x^2 \xrightarrow{\text{deriv}} du = 2x\,dx$

$dv = x\,e^{x^2}dx$ $\quad v = ??\ = \frac{1}{2}e^{x^2}$

Anti-deriv

$\int xe^{x^2}dx = \frac{1}{2}\int e^u du$

$u = x^2$ $\quad = \frac{1}{2}e^u$

$du = 2x\,dx$ $\quad = \frac{1}{2}e^{x^2}$
