

Math 144 - Final Exam Review

Exam 1 Problems

~~Final Exam~~ 16 problems @ 10 pts each
140 pts = 100%

- 4 problems from each exam

- Final: Next Wed @ 5⁴⁰ pm

1) Match the functions to their graph.

a) $f(x) = x^3 - x$



b) $f(x) = 1/(x - 1)$

c) $f(x) = 3^{-x}$

d) $f(x) = \log_2(x)$

e) $f(x) = -x^2 + 1$

f) $f(x) = 2x - 1$

Note:  = will be an exam
  = will not be an exam
 Study the rest as well because they may be an exam

2) For $f(x) = x^2 - x$ find and simplify ...

a) $f(3)$

b) $f(s)$

c) $f(a + h)$

d) $f(x^2 - x)$

3) Simplify the given expressions ...

a) $x(x^2 - 1)(x + 1)$

b) $(x + 2)/(x^2 - 4)$

4) Simplify the given expressions ...

a) $x^3(x^2)^3$

b) $\log_2(x + 1) - \log_2(x^2 - 1)$

c) $2^x 2^{-(x^2+x)}$

~~5) Find the given limits ...~~

a) $\lim_{x \rightarrow 2} x^2 - x + 3$

b) $\lim_{x \rightarrow 0} 3^x - x^3 + 1$

c) $\lim_{x \rightarrow 1^+} 1/(x - 1)$

~~6) Find the given limits ...~~

a) $\lim_{x \rightarrow +\infty} (2x^2 - 1)/(3x^2 + x + 1)$

b) $\lim_{x \rightarrow -\infty} 2^x$

7) For the given graph where is it continuous? Where is it discontinuous?

8) Solve $(x+1)/(x^2-x-6) > 0$

$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

9) Use the limit definition of the derivative to find $\frac{d}{dx}(x^2+x)$.

$$= \lim_{h \rightarrow 0} \frac{[(x+h)^2 + (x+h)] - [x^2 + x]}{h}$$

10) Find the given derivatives ...

- a) $\frac{d}{dx}(x^2 + 4x + 5)$
- b) $\frac{d}{ds}(s^{1/2} + s)$
- c) $\frac{d}{dt}(t^{2/3} - 1/t)$
- d) $\frac{d}{dx}(x(x+2))$
- e) $\frac{d}{ds}((s+1)(2s-3))$

11) Find the given derivatives ...

- a) $\frac{d}{dx}((x^2-9)/(x+3))$
- b) $\frac{d}{dx}((x^2+4x+5)/(x^{1/2}))$

$$\frac{d}{dx} \left[\frac{x^2-9}{x+3} \right] = \frac{d}{dx} \left[\frac{(x+3)(x-3)}{(x+3)} \right] = \frac{d}{dx} [x-3] = 1$$

12) If you measure the side of a square to be 4 in ± 0.1 in. What is the approximate error in area?

13) If your cost function is $C(x) = 3x + 4$ and your revenue function is $R(x) = x(10 - 0.5x)$. What is the marginal cost, marginal revenue, and marginal profit functions? Find the marginal profit at $x = 8$ and interpret your result.

take the derivative

Exam 2 Problems

problem covering the basic rules

1) Find the given derivatives (do not simplify your answer) ...

- a) $\frac{d}{dx}(x^2 + 4x + 5) = 2x + 4$
- b) $\frac{d}{dt}(t^{2/3} - 1/t + 3t) = \frac{2}{3}t^{-1/3} + t^{-2} + 3$

2) Find the given derivatives (do not simplify your answer) ...

- a) $\frac{d}{dx}(e^x + \ln(x) + 5 - x^{1/2}) = e^x + \frac{1}{x} - \frac{1}{2}x^{-1/2}$
- b) $\frac{d}{dt}(3e^t - 4\ln(t) - 7)$

3) Find the given derivatives (do not simplify your answer) ...

- a) $\frac{d}{dx}((x^2-9)(\ln(x)))$

Product Rule

b) $\frac{d}{dx}((x^2 + 4x + 5)(x^{1/2} - e^x + 3))$

4) Find the given derivatives (do not simplify your answer) ...

a) $\frac{d}{dx}\left(\frac{x^2 + 4x + 5}{x^3 - x^2}\right)$

b) $\frac{d}{dt}\left(\frac{t^3 + e^t + t}{\ln(t) - t^2 + 1}\right)$

Quotient Rule

~~5) Find the given derivatives (do not simplify your answer) ...~~

a) $\frac{d}{dx}((x^2 + 4x + 5)^{2/3})$

b) $\frac{d}{dt}(\ln(t^3 - 2t + 4))$

6) Find the given derivatives (do not simplify your answer) ...

$\frac{d}{dt}\left(t^{1/2} - \ln(t) + \frac{t^2 + 1}{2t + 3}\right)$

7) Find the given derivatives (do not simplify your answer) ...

$\frac{d}{dt}(te^{2t} - e^{2t})$ — uses chain rule
uses prod. rule

8) Use implicit differentiation to find $\frac{dy}{dx}$.

$3y^2 - 2xy + x^3 = x - 2y$

9) Use implicit differentiation to find $\frac{dy}{dx}$.

$y - xy^2 + x^2 + 1 = 0$

10) Find $\frac{dy}{dt}$ if $\frac{dx}{dt} = 2$, $x = 1$, $y = 3$, and $x^2 + y^2 = x + y$.

11) Find $\frac{dR}{dt}$ if $\frac{dx}{dt} = 500$, $x = 2000$, and $R = 10x - 0.001x^2$.

Implicit Derivatives
(see problem)

~~12) Find relative rate of change of $f(x)$ when $f(x) = 4x^3(e^x + \ln(x))$.~~

~~13) Find the Elasticity of Demand, $E(p) = \frac{-pf'(p)}{f(p)}$, if the price p and the demand x are related by the price-demand equation:~~

$x = \sqrt{200 - 2p^2}$

Exam 3 Problems

1) Find the given limits ...

a) $\lim_{x \rightarrow 1} \frac{e^x - e}{2x - 1}$

b) $\lim_{x \rightarrow 0} \frac{x - 1}{\ln(x^2)}$

$\frac{e^1 - e}{2(1) - 1} = \frac{0}{1} = 0$
 $\frac{0 - 1}{\ln(0^2)} = 0$

2) Find the given limits ...

a) $\lim_{x \rightarrow 0} \frac{3x + 1 - e^{2x}}{x^2}$

b) $\lim_{x \rightarrow 1} \frac{2x^3 - 3x^2 + 1}{2 - 3x + x^3}$

$\frac{3(0) + 1 - e^{2(0)}}{0^2} = \frac{0}{0}$ (undefined!)
 L'Hopital's $\lim_{x \rightarrow 0} \frac{3 - 2e^{2x}}{2x} = \frac{3 - 2}{0} = \frac{1}{0} = \infty$

3) Find the given limits ...

a) $\lim_{x \rightarrow 0^+} x \ln(x)$

HINT: $x \ln(x) = \frac{\ln(x)}{1/x}$

b) $\lim_{x \rightarrow 0^+} \sqrt{x} \ln(x)$

4) Given the information below when is $f(x)$ increasing? decreasing? concave up? concave down?

$f(x) = \frac{1}{4}x^4 + \frac{1}{3}x^3 - 2x^2 - 4x$

$f'(x) = (x^2 - 4)(x + 1)$

$f''(x) = 3x^2 + 2x - 4$

5) Find the first and second derivatives for the given $f(x)$. Simplify your answers.

$f(x) = x^3 - x^2 + x - 1$

6) Find the first and second derivatives for the given $f(x)$. Simplify your answers

$f(x) = (x^2 - 1)e^{2x}$

7) For $f(x) = \frac{x-1}{x-2}$ find the domain of f . Find the intercepts. Find the asymptotes.

8) For $f(x) = \frac{x-1}{x-2}$ find the first derivative f' . Find the intervals when f is increasing or decreasing. Find all relative maxs and mins.

9) For $f(x) = \frac{x-1}{x-2}$ find the second derivative f'' . Find the intervals when f is concave up or concave down. And use your solutions from problems 7 and 8 to sketch the function.

10) Find the absolute maximum and absolute minimum of $f(x) = x^3 + 3x^2 - 9x - 7$ on $[-2, 2]$.

11) Find the absolute maximum and absolute minimum of $f(x) = 9 - x^2$ on $[-1, 2]$.

12) Maximize $A = xy$ subject to $2x + y = 80$ and if x must be positive.

13) Maximize $R(x) = (80 - x)(100 + x)$ on $0 \leq x \leq 80$

$R' = 0$ find critical points
steps

step 4
| test for max |

Exam 4 Problems

1) Evaluate the given integral ...

a)

$$\int_1^2 x^3 - x^{1/2} + e^x - \frac{1}{x} dx$$

b)

$$\int (2x - 1)(x^{1/2} + x^2) dx$$

c)

$$\int \frac{(x+1)(x-1)}{x^2} dx$$

2) Evaluate the given definite integral

a)

$$\int_0^1 (2x^3 - x^2 + 3) dx$$

b)

$$\int_0^4 (x^{1/2} + 2) dx$$

3) Use substitution to evaluate the given integral

a)

$$\int 2x(x^2 - 3)^{1/2} dx$$

b)

$$\int x^{1/2}(x^{3/2} + 1)^3 dx$$

c) (Problem 3 Continued Use substitution to evaluate the given integral)

$$\int x\sqrt{2x-1} dx$$

4) Use substitution to evaluate the given definite integral

$$\int_0^1 x \sqrt{1+3x^2} dx$$

5) Use integration by parts to evaluate the given integral ...

a)

$$\int x \ln(x) dx$$

b)

$$\int x e^{-x} dx$$

6) Use integration by parts to evaluate the given definite integral

$$\int_1^e \ln(x) dx$$

7) Solve the differential equation

$$\frac{dy}{dt} = k(M - y)$$

for k and M just being constants and initial value $y(0) = 0$.

$$\int \frac{1}{M-y} dy = \int k dt$$

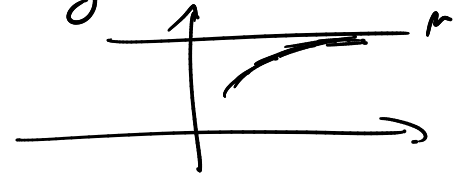
$$-\ln(M-y) = kt + C$$

$$\ln(M-y) = -kt - C$$

$$M-y = e^{-kt-C} = e^{-kt} \cdot e^{-C}$$

$$M-y = C e^{-kt}$$

$$y = M - C e^{-kt}$$



~~8) Use substitution to evaluate the given definite integral~~

$$\int_0^1 \frac{3x-3}{x^2-2x+3} dx$$

~~9) Use integration by parts to evaluate the given definite integral~~

$$\int_1^2 x^3 e^{x^2} dx$$

10) Find the area bounded by $f(x) = x^2 - 1$ and $g(x) = \frac{1}{2}x + 1$ for $-2 \leq x \leq 1$.

11) Find the consumers' surplus and the producers' surplus at the equilibrium price level for the demand equation $p = D(x) = 25 - 0.004x^2$ and the supply equation $p = S(x) = 5 + 0.004x^2$. Include a graph that identifies the surplus'.