Math 321

Proofs: $(\square \to \Delta)$ $\equiv (\neg\Delta \to \neg\square)$

3 techs
1. Direct
2. Contraposition
3. contradiction → show $(\square \wedge \neg\Delta) \equiv F$

Cases: $(\square_1 \vee \square_2) \to \Delta$

$$\equiv \underbrace{(\square_1 \to \Delta)}_{\text{case 1}} \wedge \underbrace{(\square_2 \to \Delta)}_{\text{case 2}}$$

Biconditional $\square \overset{\text{if and only if}}{\longleftrightarrow} \Delta$ (or $\square \equiv \Delta$)

show: $\underbrace{(\square \to \Delta)}_{\text{case 1}} \wedge \underbrace{(\Delta \to \square)}_{\text{case 2}}$

ex HW 2) n is even if and only if $7n+4$ is even

case 1 (n is even → $7n+4$ is even) use direct

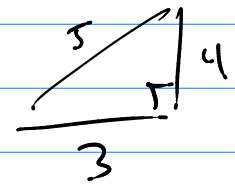
case 2 ($7n+4$ is even → n is even) Direct?

contraposition: (n is odd → $7n+4$ is odd)

contradiction show ($7n+4$ is even and n is odd) $\overset{?}{\equiv} F$

integer only algebra (ex) $\boxed{3^2 + 4^2 = 5^2}$

$x^2 + y^2 = z^2$



$x^3 + y^3 + z^3 = n$ ← integer

for $n = 1, 2, 3, \ldots, 100$

Existential Proof $\exists x\, P(x)$

the element that makes $P(x)$ true is the witness.

(1) Constructive proof: find the witness

(ex) $x^3 + y^3 + z^3 = 33$

$$8866128975287528^3 + (-8778405442862239)^3 + (-2736111468807040)^3 = 33$$

$P(x)$: "there is a square beside a cube"

Squares: 0, 1, 4, 9, 16, 25, ...

Cubes: 0, 1, 8, 27, ...

witness!

(2) <u>non-constructive</u> show witness exists w/o naming it.

$(\text{irrational})^{\text{irrational}}$ <u>is</u> rational

(ex) $\sqrt{2}^{\sqrt{2}}$ know either → rational or (exclusive or) → irrational

<u>case 1</u> $\boxed{\sqrt{2}^{\sqrt{2}}}$ is rational (then it is my witness)

witness

or

<u>case 2</u> $\sqrt{2}^{\sqrt{2}}$ is irrational

then $\boxed{\left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}}} = \sqrt{2}^{2} = \frac{2}{1}$ is <u>rational!</u>

witness

$\boxed{\text{Special Case}}$ Vacuous proof or trivial proof

<u>Vacuous:</u> all my grandkids are married $\equiv T$

if $\boxed{\text{Mark has grandkids}}$ → they are married

F

<u>trivial</u> if $\boxed{F / T}$ → $(1 = 1)$

T

Ahh!

$1 = 3$

$-2 \quad -2$

$-1 = 1$

$1 = 1$

$1-2 \qquad 3-2$

$1 = 3 \qquad 1 = 1$

$x = 1$

$x+2 = 3+2$

Exam 1 11 probs @ 10 pts each

100 pts = 100%

Prop. Logic

1. Write the truth table everyone should know.
2. eng → symbols + operators give truth table
3. Sym → eng
4. logic puzzle (truth teller / liar)

Logically Equiv.

5. Show $\square \equiv \Delta$ by truth table
6. use the logical equiv to simplify an expression.

Quantification:

7. symbols → truth value.
8. eng ⇄ sym

Proofs:

9 direct proof

10 contraposition proof

11 $\sqrt{2}$ is irrational