

# Math 321

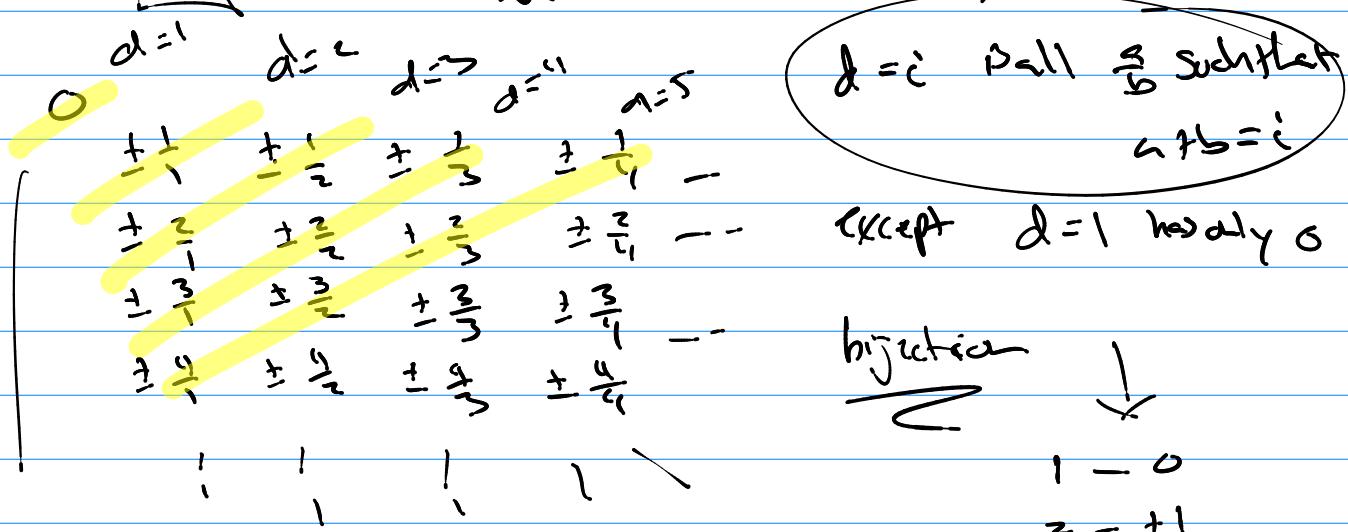
	$x_3$	$x_2$		$x_3$	$x_2$
HWS	1.1	$3 \rightarrow 1$		1.2	$6 \rightarrow 7$
	2.1	$3 \rightarrow 5$		2.2	$6 \rightarrow 8$
	3.1	$2 \rightarrow 6$		3.2	$6 \rightarrow 4$

↑

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HWS

Prove  $\mathbb{Q}$  is Countable (last class)



$\mathbb{Q}^2$

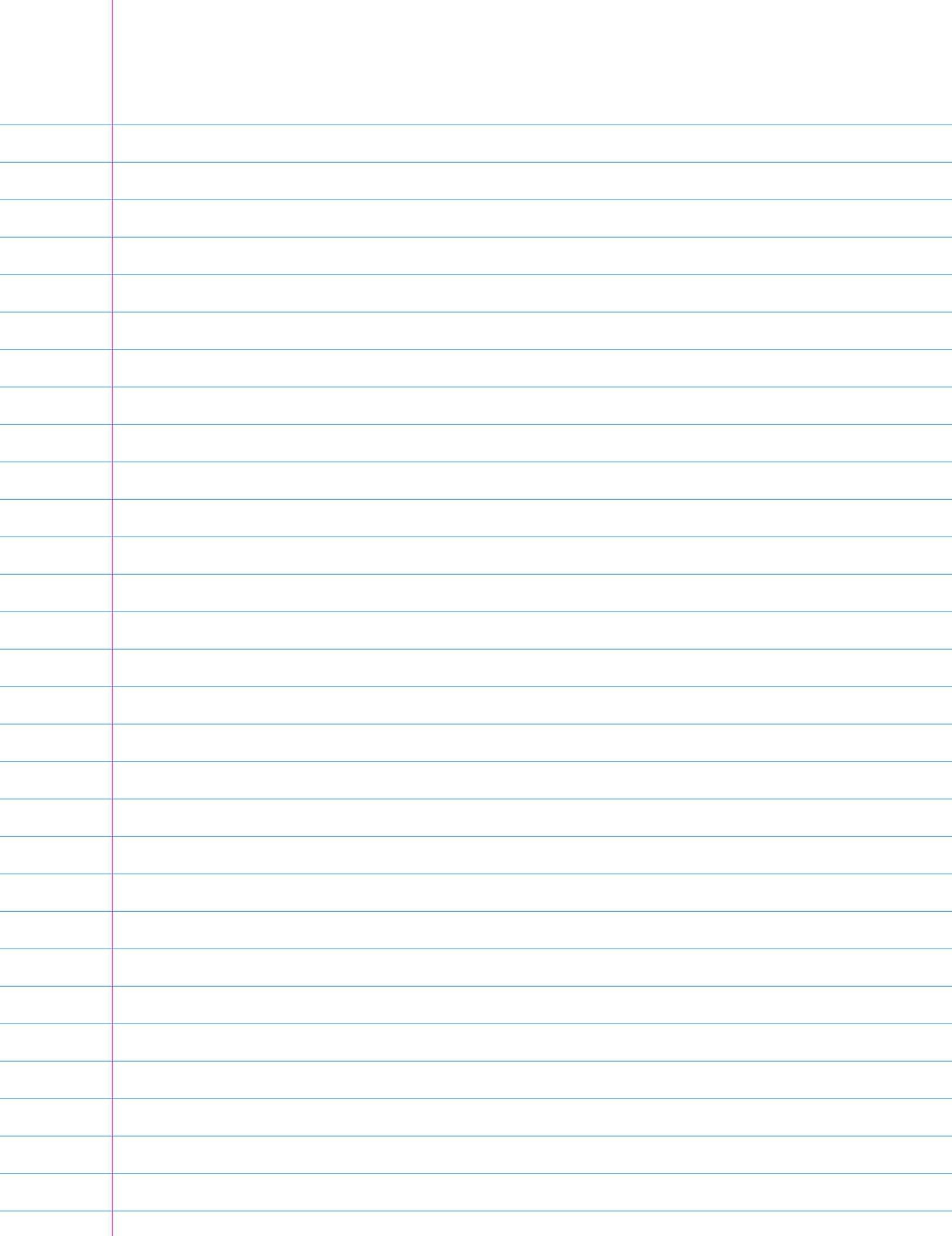
$a.b$

$$P = \{P_1, P_2, P_3, \dots\}$$

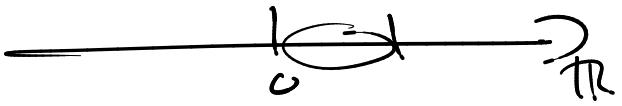
form  $(P_a)^b$

$$3.5 \rightarrow \text{form} = 5^5$$

7<sup>102</sup>



Prove  $\mathbb{R}$  is uncountable



PF

It's enough to show  $\mathbb{R}$  from 0 to 1 is uncountable

Assume

$\mathbb{R}$  from 0 to 1 are countable we have bijection

$$r^* = 0.d_1 d_2 d_3 d_4 \dots$$

$$1 - r_1 = 0.d_4 d_5 d_6 d_7 \dots$$

$$2 - r_2 = 0.d_2 d_3 d_4 d_5 \dots$$

$$3 - r_3 = 0.d_3 d_4 d_5 d_6 \dots$$

$$4 - r_4 = 0.d_4 d_5 d_6 d_7 \dots$$

⋮

dig are decimal  $\{0, 1, 2, \dots, 9\}$

b/c terminating decimal have two forms.

$$\rightarrow \frac{1}{4} = 0.\overline{250} = 0.249999 \dots$$

$$\frac{1}{5} = 0.\overline{1} = 0.099999 \dots$$

so remove all  $\overline{1}$  versions.

Now each  $r_i$  is uniq.

(Ex)  $0.\overline{133333} \stackrel{a}{=} 0.133333 \dots \rightarrow a \neq b$

$$0.\overline{133333} \stackrel{b}{=} 0.133333 \dots \rightarrow b$$

and all reals from 0 to 1 are here

Now consider  $r^* = 0.d_1 d_2 d_3 d_4 \dots$

$d_i$  for  $r^*$  are either 1 or 2.

$\forall i$  if  $d_{ii} = 1$  but  $d_i = 2 \quad r^* \neq r_i$   
 $d_{ii} \neq 1$  but  $d_i = 1$

so  $r^*$  is between 0,1 my list has all reals

between 0 and 1 and my list doesn't have it<sup>+</sup>

$\rightarrow$  always false  $\}$   
~~2~~

$$|\mathcal{P}| \neq |\mathcal{N}_0| \quad |\mathcal{TR}| = |\mathcal{N}_1|$$

$$|\mathcal{S}| = \langle \rangle^2, \dots, \boxed{\mathcal{N}_0, \mathcal{N}_1, \dots} \circ$$