

# Math 321

$$f_0 = 0 \quad f_1 = 1$$

$$f_n = f_{n-1} + f_{n-2} \quad n=2, 3, 4, \dots$$

1st class

$$f_n = \alpha_1 \left( \frac{1+\sqrt{5}}{2} \right)^n + \alpha_2 \left( \frac{1-\sqrt{5}}{2} \right)^n$$

$$n=0$$

$$n=1$$

$$\begin{cases} 0 = \alpha_1 + \alpha_2 \\ 1 = \alpha_1 \left( \frac{1+\sqrt{5}}{2} \right) + \alpha_2 \left( \frac{1-\sqrt{5}}{2} \right) \end{cases}$$

$$\begin{cases} (\alpha_1 + \alpha_2 = 0) \quad (- (1+\sqrt{5})) \\ (1+\sqrt{5})\alpha_1 + (1-\sqrt{5})\alpha_2 = 2 \end{cases}$$

$$-(1+\sqrt{5})\alpha_2 + (1-\sqrt{5})\alpha_2 = 2$$

$$-2\sqrt{5}\alpha_2 = 2 \quad \alpha_2 = -\frac{1}{\sqrt{5}} \quad \alpha_1 = \frac{1}{\sqrt{5}}$$

$$\text{So } f_n = \underbrace{\frac{1}{\sqrt{5}} \left( \frac{1+\sqrt{5}}{2} \right)^n}_{\alpha_1} + \underbrace{\frac{1}{\sqrt{5}} \left( \frac{1-\sqrt{5}}{2} \right)^n}_{\alpha_2}$$

$$\text{(Ex)} \quad f_{100} = \frac{1}{\sqrt{5}} \left( \frac{1+\sqrt{5}}{2} \right)^{100} - \frac{1}{\sqrt{5}} \left( \frac{1-\sqrt{5}}{2} \right)^{100}$$

$$f_0 = 0, f_1 = 1, f_2 = 1, f_3 = 2, \dots$$

$$\begin{aligned} \frac{2}{(1+\sqrt{5})(1-\sqrt{5})} &= \frac{2(1-\sqrt{5})}{1-5} \\ &= -\frac{(1-\sqrt{5})}{2} \end{aligned}$$

(LK)

walk up  $n$ -steps and you can take 1 step = "left leg"  $\rightarrow$  "right leg"

2 steps = left leg

~~rec. relation~~

$$a_n = \underbrace{2 \cdot a_{n-1}}_{\text{take 1 step}} + \underbrace{1 \cdot a_{n-2}}_{\text{take 2 steps}}$$

Base:

$$a_0 = 1$$

$$a_1 = 2a_0 + \cancel{a_1} = 2$$

$$a_2 = 2a_1 + \cancel{a_0} = 2(2) + 1 = 5$$

$$a_3 = 2a_2 + a_1 = 2 \cdot 5 + 2 = 12$$

$$a_4 = 2a_3 + a_2 = 2 \cdot 12 + 5 = 29$$

Seq 1, 2, 5, 12, 29, ..

$$a_0 = 1 \quad a_1 = 2$$

Solve:

$$a_n = 2a_{n-1} + a_{n-2}$$

$$\underline{\underline{a_n = f(n)}}$$

techniques:

forward iteration

$$a_0 = 1$$

$$a_1 = 2$$

$$a_2 = 2 \cdot 2 + 1 = 2^1 + 1$$

$$a_3 = 2(2 \cdot 2 + 1) + 2$$

$$= 2^3 + 2 \cdot 2 + 1$$

$$a_4 = 2(2^3 + 2^2) +$$

$$(2^2 + 1)$$

$$= 2^4 + 2^3 + 2^2 + 1$$

If  $a_n$  rec. relation is  $\leftarrow$  linear homogeneous rec. relation  
 of deg K with constant coeff.

$$\rightarrow \underline{a_n = r^n}$$

(ex)

$$a_n = 2a_{n-1} + 1 \cdot a_{n-2}$$

$\therefore r^n$   
 This is sol.

$$r^n = 2r^{n-1} + r^{n-2}$$

$$r^2 = 2r + 1$$

$$\Rightarrow r^2 - 2r - 1 = 0$$

$$r = \frac{2 \pm \sqrt{4+4}}{2} = \frac{2 \pm \sqrt{8}}{2} = 1 \pm \sqrt{2}$$

$$r = 1+\sqrt{2}$$

$$r = 1-\sqrt{2}$$

$$a_n = [2\sqrt{2}(1+\sqrt{2})^n + 2\sqrt{2}(1-\sqrt{2})^n]$$

or  $a_0 = 1, a_1 = 2$  to find  $\alpha_1, \alpha_2$

$$\begin{cases} 1 = \alpha_1 + \alpha_2 \\ 2 = \alpha_1(1+\sqrt{2}) + \alpha_2(1-\sqrt{2}) \end{cases}$$

Solve!



Exan 3) 11 probos  $\subset$  10 pts  $\rightarrow$  10 probos - 100%

Countability (3 probos)

- ① grand hotel problem.
- ② prove ① is countable
- ③ prove TR is uncountable

Seq's / Series (5 probos)

① (parts) rule  $\rightarrow$  seq

(ex)  $\{2n+3 \quad n=3,4,5,6, \dots\}$   
seq: 7, 9, 11, ...

(ex)  $a_0 = 1 \quad a_1 = -2$

$$a_n = \underline{\underline{a_{n-1} - 2a_{n-2}}}$$

seq: 1, -2, 11, 0, 8, 8, ...

② Seq  $\rightarrow$  rule

atuch

3, 7, 11, 15, ...

$\{3 + 4n^3 \quad n=0, 1, 2, \dots\}$

③ Show (prove)  $\sum_{k=0}^n ar^k = a \left\{ \frac{r^{n+1}-1}{r-1} \right\}$  p71

④ calculate some sums (know  $\sum$  rules)

$$\sum_{k=3}^{10} k^2 - 1 \quad \sum k, \sum k^2, \text{etc}$$

$$\sum_{k=2}^{14} 3 \cdot \left(\frac{1}{2}\right)^k$$

⑤ Given an open formula (rec. relation)  
find closed formula using forward/backward iteration.

Induction: (3 proofs)

① Proof using equalities

(ex)  $1 + 2 + \dots + n = \frac{n(n+1)}{2}$

② Proof using inequalities

③ Money make by \$k, \$l

(ex  $\{\$3, \$5\}$  can make \$8, \$9, \$10, ... amounts)

(ex  $\{\$5, \$7\}$  can make \$24, \$25, \$26, ...)