⊙ = on test (study every thing else)
X = not on test

16 probs @ 10pts (150 = 100%)

① 4 probs / exam

② % final → replace your lowest exam %

ex

1	2	3	4	Final
~~56%~~ 76%	70%	65%	75%	76%

Math 321 ... Final Exam Review

Exam 1 Problems

1) Construct the truth table everyone should know.

p	q	
T	F	
F	T	
F	F	
T	T	

2) Express "For the mouse to defeat the cat it is sufficient that the mouse drinks lots of coffee" using propositional symbols and logical operators. Then contruct a truth table for your compound proposition.

eng ⇄ symbols + truth table

3) Express *paper* $\wedge$ (*quiet* $\rightarrow$ *paper*) as a sentence if "paper" means "I crushed my paper exam" and "quiet" means "The room is quiet". Also state when the given compound proposition would be false.

X 4) On the island of truth tellers and liars you come upon two people. The first says "If 8 is an odd number, then I am a truth teller". And the second says "The first person is a liar". Determine whether each person is a truth teller or a liar.

5) Use a truth table to show that the statements $[(p \rightarrow q) \wedge (p \rightarrow r)]$ ~~and~~ $\leftrightarrow$ $[p \rightarrow (q \wedge r)]$ are logically equivalent.

is a tautology

Know!

6) Use logical equivalences to show that $(p \wedge q) \rightarrow p$ is a tautology.

ex

$$\equiv \neg(p \wedge q) \vee p$$
$$\equiv (\neg p \vee \neg q) \vee p \equiv \neg p \vee p \vee \neg q$$
$$\equiv T \vee \neg q \equiv T$$

X 7) Determine the truth value of the each of these statements if the domain consists of all integers ...

a) $\exists n(n^3 = -1)$

b) $\forall n(3n < 4n)$

c) $\forall x \forall y((x^2 = y^2) \rightarrow (x = y))$

8) a) Let $S(u)$ mean that "u is silly," $F(v)$ mean that "v is fast," and $B(a,b)$ mean that "a has beat b in a race", where the universe of discourse for every variable consists of all children. Express $\exists x(F(x) \wedge \forall y(S(y) \rightarrow B(x,y)))$ by a simple English sentence.

b) Use quantifiers and propositional functions to express "Every silly kid has beat a fast kid in a race".

9) Prove if a is an odd number then $a^2 + 3$ has a factor of 4.

Direct proof:

assume $a = 2k+1, \; k \in \mathbb{Z}$

$a^2 + 3 = (2k+1)^2 + 3 =$

10) For the integers 2,3,4,5,... Prove: if $n^2 < 2^n$, then $n > 4$.

Contraposition

$\boxed{n = 2,3,4 \rightarrow n^2 \geq 2^n}$

Direct

11) Prove that $\sqrt{2}$ is irrational. (Include the proof of the needed lemma)

lemma: a^2 even $\rightarrow$ a even (prove)

and then prove $\sqrt{2}$ is irrational.

$2 = \frac{a^2}{b^2}$

$2b^2 = a^2$

Exam 2 Problems

1) a) Given $a, b, c, m,$ and n are integers, Show that if $a|b$ and $a|c$, then $a|mb + nc$.

1) b) Given $a, b,$ and c are integers, Show that if $a|b$ and $b|c$, then $a|c$.

$p|q$ is $\boxed{p \cdot n = q, \; n \in \mathbb{Z}}$

2) a) Find $-25 \operatorname{div} 7$ and $-25 \bmod 7$

b) Find $25 \operatorname{div} 7$ and $25 \bmod 7$

3) List two negative integers and two positive integers that are congruent to -13 modulo 9.

$? = -13 \text{ div } 9$

$? = -13 \bmod 9$

4) Evaluate $(2^{502} + 71)^{100} \bmod 3$.

$2^{502} \bmod 3 = (2^2)^{251} \bmod 3 = 1$

$(1+71)^{100} \bmod 3$

$(72)^{100} \bmod 3 = 0$

5) Solve the congruence equations ...

a) $-2x + 13 \equiv 4 \pmod 7$

b) $3x + 28 \equiv 12(5 + x) \pmod 5$

6) Prove that there are infinitely many primes.

7) Find the prime factors, the gcd, and the lcm of 140 and 75 using prime factorization. Don't multiply out the product of primes for your answers.

8) Find the gcd of 140 and 75 using Euclid's Algorithm.

9) Using Euclid's Algorithm find the gcd of 87 and 33 and then write it as $gcd(87, 33) = s * 87 + t * 33$.

10) Given the affine-shift function: $f(p) = (5p + 11) \bmod 13$ find the decryption function $f^{-1}(c)$.

11) Given the public key of $e = 5$ and $n = 221$ of a public key encryption find the private key.

Exam 3 Problems

1) Suppose that Hilbert's Grand Hotel is fully occupied, but you need to empty all the odd numbered rooms because they smell oddly and need maintenance. As the manager, what instructions would you give to the guests to move them around so they all still have a room?

2) List the sequences ...

a) List the first 5 terms of the sequence $a_0 = 1, a_1 = 0$ and $a_n = 3a_{n-1} - a_{n-2}$ for $n = 2, 3, 4, \ldots$

b) List the first 5 terms of the sequence $\{3 + 2^{n-1}\}$ for $n = 1, 2, 3, \ldots$

3) Find some rule to generate the given sequences ...

a) 0, 3, 8, 15, 24, 35, ...

b) -2, 2, 6, 10, 14, 18, ...

c) -2, 1, -1, 0, -1, -1, -2, -3, -5, -8, ...

4) Prove the rationals are countable.

→ you get to pick which one to do.

5) Prove the reals are uncountable.

6) Prove the formula for the geometric sum ...

$$\sum_{k=0}^{n} ar^k = a\left[\frac{r^{n+1}-1}{r-1}\right]$$

7) Find the value of the given sum (you do not have to simplify your answer) ...

$$\sum_{k=3}^{111} 2k^3 - 3$$

8) Use forward or backward iteration to find a closed function for the recurence relation $h_1 = 1$ and $h_n = 3h_{n-1} + 1$.

9) Prove $1^3 + 2^3 + ... + n^3 = (\frac{n(n+1)}{2})^2$ using induction.

10) Prove $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots + \frac{1}{n^2} \leq 2 - \frac{1}{n}$ using induction.

11) Prove that you can form any dollar amount from 18 dollars on using only 4-dollar and 7-dollar bills.

Exam 4 Problems

Note: For each problem do not simplify your arithmetic solutions.

1) a) For class you must wear a tie. You have 4 red ties, 1 black tie, 3 green ties, and 3 bow ties. How many choices of neck-wear do you have?

$4+1+3+3$

b) If also you have three pairs of shoes, how many individual items do you now have?

$4+1+3+3+3\cdot 2$

2) a) Your wardrobe consists of 5 shirts, 3 pairs of pants, and 17 bow ties. How many different outfits can you make?

$5\cdot 3\cdot 17$

b) If you also have three pairs of shoes, how many outfits can you now make?

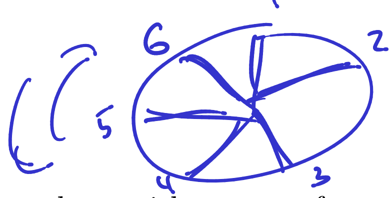
$5\cdot 3\cdot 17\cdot 3$

3) Your movie collection consists of 5 action films, 7 horror films, 10 love stories, and 6 romatic comedies. If you first want to watch an action film or love story, and then followed by a horror film, how many choices do you have?

4) In a recent survey, 30 students reported whether they liked their potatoes Mashed, French-fried, or Twice-baked. 15 liked them mashed, 20 liked French fries, and 9 liked twice baked potatoes. Additionally, 12 students liked both mashed and fried potatoes, 5 liked French fries and twice baked potatoes, 6 liked mashed and baked, and 3 liked all three styles. How many of the students did not like potatoes? Demonstrate your answer with a Venn Diagram and explain why your answer is correct.

$P(20,6) = \frac{20!}{14!}$

5) How many ways can you pick 6 people from a group of 20 ~~to sit at~~ a round table if you consider that having the same people on your right and/or left is the same seating arrangement?



overcount

$\frac{20!/14!}{6\cdot 2}$

6) Seven guys and ten girls try out for your e-sports team. How many ways to choose 5 people if at

least 2 of them must be guys?

7) You have 10 students and 17 chairs. How many functions from students to chairs are there? How many one-to-one functions from students to chairs? Explain why your answers are correct.

S_1 C_1

S_2 C_2

$\vdots$ $\vdots$

S_{10} C_{17}

$17 \cdot 17 \cdots 17 = 17^{10}$

$17 \cdot 16 \cdots$ $P(17,10) = \frac{17!}{7!}$

8) What is the 15th term of $(3x - \frac{2}{x^3})^{101}$?

9) Use the binomial theorem to show $C(n,0) + C(n,1) + \ldots + C(n,n) = 2^n$

10) Prove $C(n+1,k) = C(n,k) + C(n,k-1)$ using a counting proof.

11) Prove $1 * C(n,1) + 2 * C(n,2) + 3 * C(n,3) + \ldots + n * C(n,n) = n2^{n-1}$ using a counting proof. Hint: choose people for a committee and a leader for that committee.