

Math 322

(Q's)

total ordering

vs

well ordering

Some subset do not have a start element.

all subsets have a start element

(ex) $\{\mathbb{Z}^+, \leq\}$ as chain $1, 2, 3, 4, 5, 6, \dots$ well ordered
 $\{\mathbb{Z}^+, \geq\}$ $\dots, 4, 3, 2, 1$ not well order

Exam 1

\mathbb{R} pebs @ 10 pts
110pts = 100%

① prove $\sqrt{2}$ is irrational (with lemma)

Ch 1

a^2 even $\rightarrow a$ even

7.1 Binary Relations and Properties

$R = \{ (a,b) \mid \text{predicate on } a, b \}$

3 probs

① Does a given R have the properties
 of ...

- ref. $\forall a (aRa)$
- irref $\forall a (a \not R a)$
- sym $\forall a \forall b (aRb \rightarrow bRa)$
- antisym $\forall a \forall b (aRb \wedge bRa \rightarrow a=b)$

$$\text{asym} \quad \forall a \forall b (a R b \rightarrow b R a)$$

$$\text{trans} \quad \forall a \forall b \forall c (a R b \wedge b R c \rightarrow a R c)$$

$$\forall a \forall b (a R b \rightarrow b R a)$$

$$\equiv \forall a \forall b (a R b \vee b R a) \equiv \forall a \forall b \neg (a R b \wedge b R a)$$

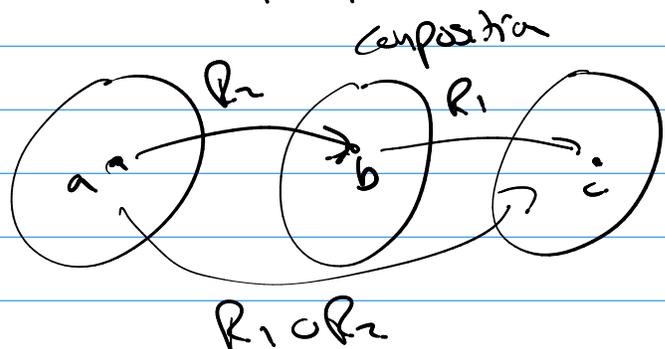
$$\equiv \neg (\exists a \exists b (a R b \wedge b R a))$$

$$\text{not (sym)} \quad \neg (\forall a \forall b (a R b \rightarrow b R a))$$

$$\equiv \exists a \exists b \neg (a R b \rightarrow b R a) \equiv \exists a \exists b (a R b \wedge b R a)$$

#2 operators: $R_1 = ?$ $R_2 = ?$ (given)

sketch $R_1 \cup R_2$, $R_1 \cap R_2$, $R_1 \circ R_2$



#3 prove R is trans (iff) $\forall n R^n \subseteq R \quad n=1,2,3, \dots$

(case #1) (\implies) induction \longleftrightarrow

(case #2) (\impliedby) Univ. instantiation (let $n=2$)

9.2 (oprob)

9.3 Representing R as set A digraph, matrices

2 probs

① represent R

② use zero-one matrices

for $R, \cup R, R \cap R,$

$\overline{M_R \cup M_R}$

$M_R \wedge M_R$

$$\left[\begin{array}{l} R \cup R \\ M_R \cup M_R \\ R \cap R \\ M_R \cap M_R \\ R \circ R \\ M_R \circ M_R \end{array} \right]$$

ex

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & \phi & 1 & 0 \\ 1 & 0 & 1 & \phi & 0 & 1 \end{array} \right] = \left[\begin{array}{l} (1 \vee 1) \vee (1 \wedge 1) \vee (1 \cap 1) \\ 1 \vee 1 \vee 1 \\ \text{I} \end{array} \right]$$

9.4

closure

ref. closure

$M_R \vee (M_R) = \square$

Sym closure

$M_R \vee M_R^T = \square$

I

trans closure

$M_{R^*} = M_R \vee M_R^{\{2\}} \vee \dots \vee M_R^{|A|}$

3 probs

- ① a) ref closure
- b) sym closure

② M_{R^*} by $M_R \vee M_R^{\{2\}} \vee \dots \vee M_R^{|A|}$

③ M_{R^*} by Warshall's

19.5

Equiv. Relations

2 probs

→ check for ref, sym, trans

① is R an equiv. relation?

ex $R = \{ (a,b) \mid a,b \text{ go to same High School} \}$

② give equiv. classes...

19.6

Partial Orders

2 probs

① Is it? (check ref, antisym, trans)

② given a poset

a) draw Hasse Diagram

b) ans questions (minimal, maximal)

Prove $\sqrt{2}$ is irrational

Step 1

lemma a^2 even $\rightarrow a$ even

pf:

by contra posita

$\rightarrow (a \text{ even}) \rightarrow (a^2 \text{ even})$

$a \text{ odd} \rightarrow a^2 \text{ odd}$

show by direct proof

$a \text{ odd} \rightarrow a = 2k+1, k \in \mathbb{Z}$

so $a^2 = (2k+1)^2$

$a^2 = 4k^2 + 4k + 1$

$a^2 = 2(2k^2 + 2k) + 1$

int odd

$2k^2 + 2k$ is an integer