O = on test

X = not on test

study all other problems

Final Exam / 16 probs @ 10 pts each / 150 pts = 100%

I'll pick 4 problems per exam to set the 16

MATH 322 - FINAL EXAM REVIEW

Exam 1 Problems

1) Is the relation R consisting of all ordered pairs (a, b) such that a and b are people and they are taking Math 322: reflexive, irreflexive, symmetric, antisymmetric, asymmetric, and/or transitive? If a property doesn't hold give a counter-example and **state the logical definitions of the properties** as you consider them.

2) Given the relation $R_1 = \{(a, b) | b = 2\, a\}$ and $R_2 = \{(a, b) | b = 3\, (a - 1)\}$ on the set of positive integers from 0 to 12. Give the list of ordered pairs for R_1 and for R_2 and find the relations $R_1 \cap R_2$ and $R_1 \cup R_2$.

3) Represent the relation $R = \{(a, a), (a, b), (a, c), (b, a), (c, a), (c, c)\}$ on the set $A = \{a, b, c, d\}$ as a digraph and a matrix.

4) Prove: R on set A is transitive if and only if $\forall n\, R^n \subseteq R$, $n = 1, 2, 3, \ldots$

5) For the set $A = \{a, b, c\}$, relation $R_1 = \{(a, a), (a, c), (b, b), (c, a)\}$, and relation $R_2 = \{(a, b), (b, a), (b, b), (c, b), (c, c)\}$. Represent the relations as matrices and then use matrix operations to find $R_1 \circ R_2$ and then $R_1 \cap R_2$.

1 prob

6) For $R = \{(a, a), (a, c), (a, d), (b, a), (b, d), (c, a), (c, d), (d, a), (d, c)\}$ on the set $A = \{a, b, c, d\}$ find the ...

a) Reflexive Closure as a matrix.

b) Symmetric Closure as a matrix.

7) For $R =$ ~~$\{(a, b), (a, d), (b, a), (b, c), (c, d), (d, c)\}$ on the set~~ $A = \{a, b, c, d\}$ find the transitive closure using Warshall's Algorithm.

8) For $R = \{(a, a), (a, b), (b, a), (b, c), (c, a)\}$ on the set $A = \{a, b, c\}$ find the transitive closure using the join of powers of M_R.

9) Show that the relation R consisting of all pairs (f, g) such that the second derivative of f and the second derivative of g are equal is an equivalence relation on the set of all polynomials with real-valued coefficients.

10) For the relation given above which functions are in the same equivalence class as $f(x) = 2\, x^2 - x$?

11) Show that $(\mathbb{Z}^+, |)$ a partial ordering.

12) Draw the Hasse diagram for $(2, 4, 5, 10, 12, 20, 25, |)$ then ...

a) State the maximal, minimal, greatest, and least elements.

b) Create a topological sort. (Note: always take the left minimal first)

Exam 2 Problems

Graphs?. → C_n, K_n, W_n, Q_n, $K_{n,m}$

1a) Name the given graph.

1b) Name the given graph.

1c) Draw an unconnected mixed graph with 6 vertices and 6 edges.

1d) Draw a pseudograph with 5 vertices that has exactly three vertices of odd degree.

2) Contruct the intersection graph for the sets for the universe of discourse being the integers from 1 to 10. The sets are $A = \{x|x > 5\}$, $B = \{x|x \text{ is even}\}$, $C = \{x|x \text{ is odd}\}$, $D = \{x|x \text{ is divisible by 2 or 3}\}$, and $E = \{x|x < 3\}$.

3) Draw the graph Q_4 and state the number of vertices, edges, and degree for each vertex. Verify that the Handshake theorem applies.

4) Draw W_5 and determine if it is bipartite. Explain and name any theorems used to determine if it is, or is not, bipartite.

5) Are the graphs isomorphic? Justify your answer. (not - isomorphic)

6) Are the graphs isomorphic? Justify your answer. (is - isomorphic)

7) Draw a directed multi-graph with 4 vertices that is weakly connected and not strongly connected.

8) For the given undirected graph find $\kappa(G)$ and $\lambda(G)$. State the vertices that make a minimal vertex cut. State the edges that make a minimal edge cut.

9) For the given image, can you trace the given graph without retracing any edges? Explain your reasoning and any used theorems.

10) State Dirac's and/or Ore's Theorems and can they be applied to Q_3? Find a Hamilton Circuit for Q_3.

11) For what values of n will $K_{n,2}$ have an Euler path? State the theorems you use.

12) Find the paths and lengths for the shortest paths between a and every other vertex in the graph.

Exam 3 Problems

1) You receive the following message via some social media application "Send the message 'I love to count in an advanced way' to 5 of your friends and you will get an A in Math 322!" If a total of 200 people send the message before it stops, how many people are in the tree? How many edges are in the tree? How many received it and did not send it out? What could you say about the height of the tree?

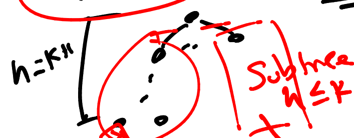
Induction: assume $l \le m^k$

Basis: $h=0$

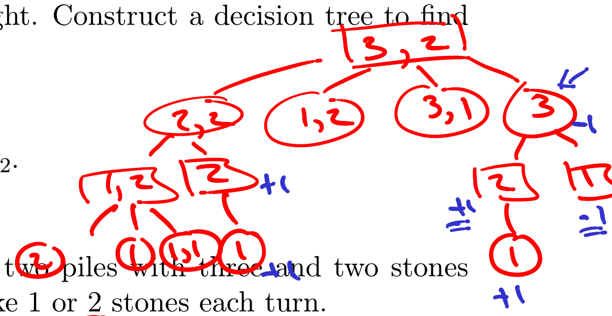
2) Prove: For an $m-$ary tree of height h with l leaves, $l \le m^h$.

show $l \le m^{k+1}$ on tree of $h = k+1$

tree $l = 1$, $m^0 = 1$, $1 \le 1$ true



3) In a best case situation, how many weighings of a balance scale are needed if given four coins you may have a counterfeit? Note: the counterfeit could be heavy or light. Construct a decision tree to find the counterfeit or determine if there is no counterfeit.

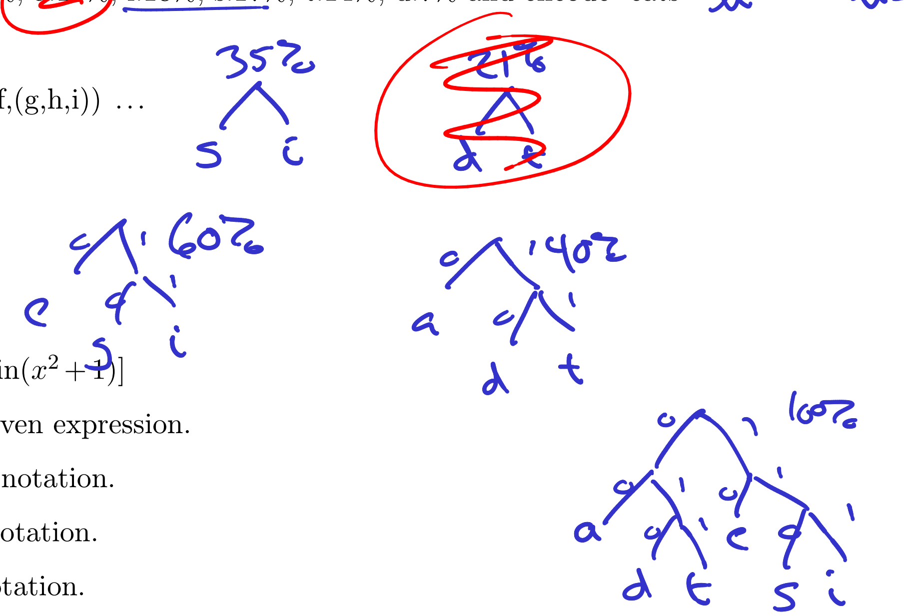


4) Create a decision tree that orders the elements of the list e_0, e_1, e_2.

5) Draw the game tree for nim if the starting position consists of two piles with three and two stones respectively [(3) , (2)]. Construct the game tree if a player can take 1 or 2 stones each turn.

6) Create the Huffman Code tree if e:25%, a:19%, i:~~18%~~, s:~~17%~~, t:~~14%~~, d:~~7%~~ and encode "eats"

left = less



7) For the in-order traversal (a,b,c),d,(e,f,(g,h,i)) ...

a) Construct the rooted tree

b) Write preorder traversal

c) Write the postorder traversal

8) For the standard expression $A_x[(3x)\sin(x^2+1)]$

a) Construct the rooted tree for the given expression.

b) Write the expression using post-fix notation.

c) Write the expression using pre-fix notation.

d) Write the expression using in-fix notation.

9) Construct the bit table for $(\overline{x+y}) \cdot z$.

a : 00

d : 010

t : 011

10) Using only the Identity, Complement, Associative, Communtative, and/or Distributive laws of a Boolean Algebra verify ...

a) $x \cdot x = x$

b) $\bar{0} = 1$.

$x = x \cdot 1 = x \cdot (x + \bar{x}) = x \cdot x + x \cdot \bar{x} = x \cdot x + 0 = x \cdot x$

$\bar{0} = \bar{0} + 0 = 1$

11) Find the sum of products for $F(x, y, z) = x \cdot (y + (y \cdot z))$ using any technique.

12) Find the product of sums for $F(x,\ y,\ z) = (x + z) \cdot y$ by using a technique you DID NOT use in problem 11.

table → sum of prod
→ prod of sums

Exam 4 Problems

1) For the grammar with $V = \{0, 1, A, B, S\}$, $T = \{0, 1\}$, and the productions $S \to 0A$, $S \to B1$, $S \to \lambda$, $A \to 0B1$, $B \to 1$, and $A \to 0$ find L(G).

2) Name the grammar type (give its type number and name) and circle the productions that prevent it from being the next type.

a) $S \to A, S \to B, S \to \lambda, A \to Sb, B \to aB, A \to a$, and $B \to b$

b) $S \to AB, A \to aA, B \to bB, A \to a$, and $B \to b$

c) $S \to AB, S \to \lambda, B \to aAb, aA \to a$, and $A \to B$

d) $S \to \lambda, S \to aA, A \to bB, B \to a$, and $A \to b$

3) Given L(G) = 01^* find a set of productions that will produce L(G).

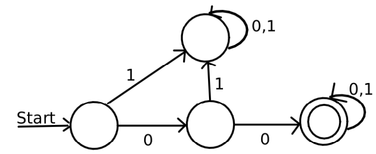
Vend? or Add?

4) Construct a finite-state machine with output that models a candy machine that accepts only dimes. Candy costs 20 cents and the machine keeps the money for any amount greater than 20 cents. The customer can push buttons to receive candy or to return pennies. ~~Represent the machine with a state table.~~

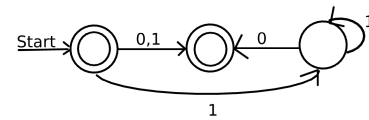
5) Construct a finite-state machine with output that delays input by two bits using 10 for the delay. Represent the machine with a state diagram.

6) Given the language L $= \{\lambda, 0, 11(0,1)^*\}$ create a finite-state automaton that recognizes the language.

7) Determine the language recognized by a given deterministic finite-state automaton.



8) Determine the language recognized by a given non-deterministic finite-state automaton.



9) Using the inductive constructions described in the proof of Kleene's Theorem, find a non-deterministic finite-state automaton that recognizes $0 \cup (01)^*$.

10) Construct a non-deterministic finite-state automaton that recognizes the language generated by the regular grammar with $V = \{0, 1, A, S\}$, $T = \{0, 1\}$, and the productions $S \to 1A$, $S \to 0B$, $S \to \lambda$, $A \to 0A$, $A \to 0$, $B \to 1B$, $B \to 0$.

11) Let T be the Turing machine defined by the five-tuples: $(s_0, 1, s_0, 1, R)$, $(s_0, 0, s_1, 1, R)$, $(s_0, B, s_1, 1, R)$, $(s_1, 1, s_1, 1, R)$, $(s_1, 0, s_2, 1, R)$, and $(s_1, B, s_2, 1, R)$. Run the Turning machine on the below initial tape, write each of the positions, and determine the tape when T halts. Does T recognize the input string? What do you think T is doing?

Inital Tape: ... B,B,B,1,0,1,0,1,1,B,B,B, ...

12) Construct a Turing machine for $f(n) = n \bmod 2$. Run your machine on the input: 1,1,1,1,1.

know $\left[\begin{array}{c} n \bmod k \\ n_1 + n_2 \\ n - k \end{array}\right]$

n_1 * n_2