

# Math 321

$P$ : "object(s) has property that I can 'test'"

$\downarrow$   
predicate

$\uparrow$   
 yes = true  
 no = false

## operate

①  $\neg P$ : "It is not the case that  $P$ "

②  $P \wedge Q$ : " $P$  and  $Q$ "     Note: and = but = yet

③  $P \vee Q$ : " $P$  or  $Q$ "

④  $P \oplus Q$ : " $P$  or  $Q$ , but not both"

⑤  $P \rightarrow Q$ : "if  $P$  then  $Q$ "

⑥ ex happy: " "

jump: " "

happy is nec. but not suff. for jump.

$\uparrow$       $\uparrow$       $\uparrow$       $\uparrow$   
 implication     conj.     implication  
 neg.

$$(jump \rightarrow happy) \wedge \neg (happy \rightarrow jump)$$

⑦  $P \leftrightarrow Q$ : " $P$  if and only if  $Q$ "

$P$	$Q$	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

truth table everyone should know

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \oplus Q$	$P \rightarrow Q$	$P \leftrightarrow Q$
T	T	F	T	T	F	T	T
T	F	F	F	T	T	F	F
F	T	T	F	T	T	T	F
F	F	T	F	F	F	T	T

Stuff based on  $P \rightarrow Q$  (implication)

①  $\neg Q \rightarrow \neg P$  called the contrapositive of original  $P \rightarrow Q$

②  $Q \rightarrow P$  called the converse of  $P \rightarrow Q$

③  $\neg P \rightarrow \neg Q$  called the inverse of  $P \rightarrow Q$

P	Q	$\neg P$	$\neg Q$	orig. $P \rightarrow Q$	contrapos. $\neg Q \rightarrow \neg P$	converse $Q \rightarrow P$	inverse $\neg P \rightarrow \neg Q$
T	T	F	F	T	T	T	T
T	F	F	T	F	F	F	F
F	T	T	F	T	T	T	T
F	F	T	T	T	T	F	F

## Applications of propositional logic

ex (jump  $\rightarrow$  happy)  $\wedge \neg$  (happy  $\rightarrow$  jump)

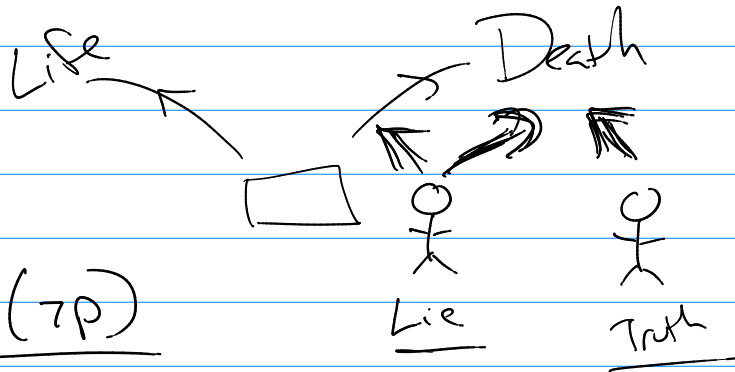
order of ops

level	op/symbols
0	grouping
1	$\neg$
2	$\wedge, \vee, \oplus$
3	$\rightarrow, \leftrightarrow$

jump	happy	jump $\rightarrow$ happy	happy $\rightarrow$ jump	$\neg$ (happy $\rightarrow$ jump)	(jump $\rightarrow$ happy) $\wedge$ $\neg$ (happy $\rightarrow$ j)
T	T	T	T	F	T
T	F	F	F	T	F
F	T	T	T	F	T
F	F	F	F	T	F

Discussion / Talking    english  $\rightarrow$  syn = truth table

Logic Puzzles



P	$\neg P$	$\neg(\neg P)$
T	F	T
F	T	F

- Knight (always truthful)
- Knave (always lie)
- Spy (does whatever he wants)

A: "C is the knave"

B: "A is the knight"

C: "I am the spy"

$\rightarrow$  C is spy or knave

(A or B) is Knight

knight  $\rightarrow$  C = knave  $\rightarrow$  B = spy