Math 321

Q's 1.1 (15e)

bear: Saw a bear
Safe: Safe to hike
berry: ripe berries

Safe, if is necessary ~~but~~ and not sufficient that (¬berry and ¬bear)

$$(\text{Safe} \to (\neg\text{berry} \wedge \neg\text{bear})) \wedge \neg((\neg\text{berry} \wedge \neg\text{bear}) \to \text{safe})$$

Propositional Logic

toys: propositions + operators: $\neg, \wedge, \vee, \oplus, \to, \leftrightarrow$

Same?

p	q	$p \to q$	$\neg p \vee q$	$(p \to q) \leftrightarrow (\neg p \vee q)$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

Def: always true : Tautology
Sometimes true : Contingency
Sometimes false :
always false : Contradiction

Def: If $\square \leftrightarrow \triangle$ is a tautology
we say $\square \equiv \triangle$ are logically equivalent.

Using "same" things.

$3(x+2) = 3x+6$

$\frac{x+2}{x^2-4} = \frac{1}{x-2}$, $x \neq -2$

$2 \cdot y = y \cdot 2$

Laws & useful logical equivalences.

(ex) $p \wedge T \equiv p$

how to show $\square \equiv \triangle$?

① truth table show $\square \leftrightarrow \triangle$ is a tautology.

(ex) $(p \wedge T) \leftrightarrow p$

p	$p \wedge T$	$(p \wedge T) \leftrightarrow p$
T	T	T
F	F	T

tautology so $(p \wedge T) \equiv p$

$p \vee F \equiv p$	identity laws.	$p \wedge F \equiv F$ Domination laws
$p \wedge T \equiv p$		$p \vee T \equiv T$

$p \wedge p \equiv p$	Idempotent laws	$p \vee \neg p \equiv T$ negation laws
$p \vee p \equiv p$		$p \wedge \neg p \equiv F$

$\neg(\neg p) \equiv p$ double negation.

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

Demorgan's laws

show table: $\neg(p \wedge q) \leftrightarrow (\neg p \vee \neg q)$ is? tautology

p	q	$p \wedge q$	$\neg(p \wedge q)$	$\neg p \vee \neg q$	( ) ↔ ( )
T	T	T	F	F	T
T	F	F	T	T	T
F	T	F	T	T	T
F	F	F	T	T	T

↑ Yes!

or by discussion

① when is left side → T? → pick one.
→ F?
find all conditions

② when is right side → T? → pick same one
→ F?
find all conditions.

③ Compare.

ex $\neg(p \vee q) \equiv \neg p \wedge \neg q$

① $\neg(p \vee q)$ when T? So $p \vee q$ is F → so p is F, q is F

② $\neg p \wedge \neg q$ when T? So $\neg p$ is T → p is F, q is F → both false
$\neg q$ is T

③ Same conditions So $\neg(p \vee q) \equiv \neg p \wedge \neg q$

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ Distributive

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

$p \wedge q \equiv q \wedge p \quad p \vee q \equiv q \vee p$ Commutative

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$p \to (q \wedge r) \equiv (p \to q) \wedge (p \to r)$

$p \to (q \vee r) \equiv (p \to q) \vee (p \to r)$

$(q \wedge r) \to p \equiv (q \to p) \vee (r \to p)$

$(q \vee r) \to p \equiv (q \to p) \wedge (r \to p)$

Associative: $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

$(p \vee q) \vee r \equiv p \vee (q \vee r)$

Conditional / biconditionals

$(p \to q) \equiv (\neg q \to \neg p)$

$(\neg p \to \neg q) \equiv (q \to p)$

$(p \to q) \equiv \neg p \vee q$

$\neg (p \to q) \equiv p \wedge \neg q$

$(p \leftrightarrow q) \equiv (p \to q) \wedge (q \to p)$

1.4 Propositional Function.

Predicate

Proposition: "Declares an object has a property"

P: "Mark eats rice"

P(x): "X eats rice" — Propositional function.

↑ what are all possible objects? (universe of discourse)

replace x → 3 ways

① pick a specific object p(mark)

Quantifier ② All

③ Some