Math 321

Q

1.3 #11 $\quad (p \wedge q) \to p$

9a

p	q	$p \wedge q$	$(p \wedge q) \to p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

1.3 prop. equiv.

11a $\quad (p \wedge q) \to p$

$\equiv \neg(p \wedge q) \vee p$

$\equiv (\neg p \vee \neg q) \vee p$

$\equiv \neg p \vee p \vee \neg q$

$\equiv T \vee \neg q$

$\equiv T$

Know: $(p \to q) \equiv \neg p \vee q$

$$\left(\boxed{p \wedge q} \to p\right) \equiv \neg \boxed{p \wedge q} \vee p$$

1.3 #15 $\quad$ is $(\neg q \wedge (p \to q)) \to \neg p$ a tautology?

① truth table? $\quad$ ② use logical equiv. $\quad$ ③ Discuss: when false?

if ans = Never

p	q	
T	T	
T	F	
F	T	
F	F	

$(\boxed{\neg q \wedge (p \to q)}) \to \neg p$

$\equiv \neg(\neg q \wedge (p \to q)) \vee \neg p$

$\equiv q \vee \neg(p \to q) \vee \neg p$ $\quad$ (*)

$\equiv \neg p \vee q \vee \neg(p \to q)$

$\equiv (p \to q) \vee \neg(p \to q) \equiv T$

$\square \to \bigcirc \equiv \neg \square \vee \bigcirc$

Compare to: (*) $\quad q \vee \neg(p \to q) \vee \neg p \equiv q \vee \neg(\neg p \vee q) \vee \neg p$

$\equiv \boxed{q \vee} (p \wedge \neg q) \vee \neg p \equiv [(q \vee p) \wedge (q \vee \neg q)] \vee \neg p$

$\equiv [(q \vee p) \wedge T] \vee \neg p \equiv [(q \vee p)] \vee \neg p \equiv q \vee T \equiv T$

1.4/1.5

$P(x)$: "object x has predicate P"

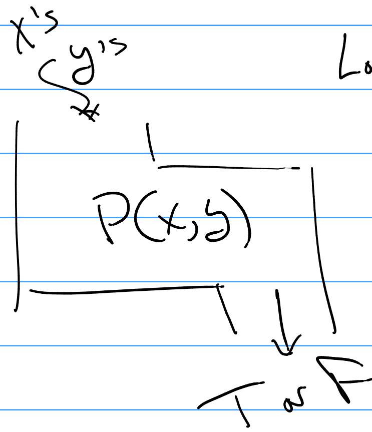
x is from a set called the universe of discourse

n-ary predicate $P(x_1, x_2, \ldots, x_n)$: "objects $(x_1, x_2, \ldots, x_n)$ have P"

x_i has its Universe of discourse (U.D.)

<u>Function:</u>

2-ary



Love(x, y): "x loves y"

U.D. of x is people

U.D. of y is food

<u>Note:</u> on U.D. if it is unstated you take a "natural" concept for it.

vs if you are told U.D. it is a <u>restricted U.D.</u>

Prop. function <u>into</u> a proposition (Binding the free variables)

1. Substitution / Evaluation.

   ex Loves(Mark, Swiss Cheese):

   "Mark loves Swiss cheese"

2. Universal Quantification

   Every object in U.D. has predicate P.

   $\forall x$ Loves(Mark, x): Mark loves all food.

<u>When false?</u> if you have <u>one</u> (or more) object where predicate is false. (Counter example)

(3) Existential Quantification:

Some object has predicate P.
one or more

$\exists p$ Loves(p, ice cream): "Someone loves ice cream"
one or more

when true? if you find the object that has predicate (called witness)

when false? no witness.

if U.D. is finite $o_1, o_2, o_3, \dots o_n$

$$\forall x P(x) \equiv P(o_1) \wedge P(o_2) \wedge \dots \wedge P(o_n)$$
$$\exists x P(x) \equiv P(o_1) \vee P(o_2) \vee \dots \vee P(o_n)$$

So $\neg \forall x P(x) \equiv \exists x \neg P(x)$ $\quad \forall x P(x)$

$\neg \exists x P(x) \equiv \forall x \neg P(x)$ $\quad \exists x P(x)$

Eng $\rightleftarrows$ sym $\quad \forall x (P(x) \Rightarrow Q(x))$ $\leftarrow$ all P are Q

$\cancel{\forall x (P(x) \wedge Q(x))}$ $\leftarrow$

$\exists x (P(x) \Rightarrow Q(x))$ $\leftarrow$

$\exists x (P(x) \wedge Q(x))$ $\leftarrow$ some P, Q object