Math 321

Q's 1.3 #17 Show $\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

by discussion:

① When is left true? → find conditions f p, q
② When is right true? → find conditions f p, q
③ Compare conditions f ① and ②

① when is $\neg(p \leftrightarrow q)$ True? ~~when~~ $p \leftrightarrow q$ is false.
only when $p \equiv T$ $q \equiv F$ and $p \equiv F$ $q \equiv T$

② when is $p \leftrightarrow \neg q$ True? when $p \equiv T$ $\neg q \equiv T$ and $p \equiv F$ $\neg q \equiv F$
so $p \equiv T$ $q \equiv F$ and $p \equiv F$ $q \equiv T$

③ Same conditions, so log. equiv.

1.4 23c "$\underbrace{\text{Someone}}_{\text{there is}}$ in class was not born in California"

Prop. Functions: Math321(s): "s is in Math 321"
BC(p): "p was born in CA"

→ U.D. of both functions is all students.

$$\exists x\,(\text{Math321}(x) \land \neg BC(x))$$

→ U.D. of both functions is students in Math 321

$$\exists x\ \neg BC(x)$$

$\exists x \left(Math321(x) \land \neg BC(x) \right)$

$\equiv \exists x, \neg \left(\neg Math321(x) \lor BC(x) \right)$

$\equiv \neg \forall x \left(\neg Math321(x) \lor BC(x) \right)$

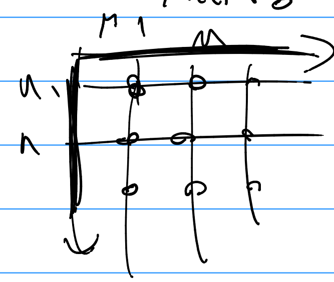
$\equiv \neg \forall x \left(Math321(x) \to BC(x) \right)$

1.5 n-ary predicates

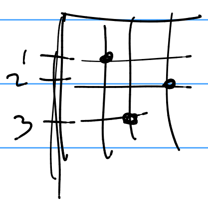
ex $P(x,y)$: "$x+y=0$"

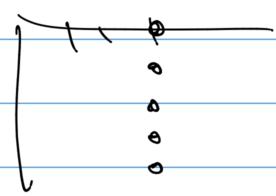
$\forall n \, \exists m \, P(n,m)$ leads to the question.. does order of quantification matter?

check: $\forall n \, \forall m \, P(n,m) \equiv \forall m \, \forall n \, P(n,m)$



$\forall n \, \boxed{\exists m} \, P(n,m) \not\equiv \exists m \, \forall n \, P(n,m)$





$\exists n \, \exists m \, P(n,m) \equiv \exists m \, \exists n \, P(n,m)$

Scope and predicates outside of scope.

Scope: $\exists x \left(P(x) \land Q(x) \right)$

$$\boxed{\begin{gathered}[\exists x \, P(x)] \land [\exists x \, Q(x)] \\ \equiv \exists s \, P(s) \land \exists r \, Q(r)\end{gathered}}$$

$$\forall x \, (P(x) \overset{\vee}{\wedge} A) \equiv (\forall x \, P(x)) \overset{\vee}{\wedge} A$$

$$\exists x \, (P(x) \overset{\vee}{\wedge} A) \equiv (\exists x \, P(x)) \overset{\vee}{\wedge} A$$

$$\forall x \, (P(x) \to A) \equiv (\exists x \, P(x)) \to A$$
$$\exists x \, (P(x) \to A) \equiv (\forall x \, P(x)) \to A$$

$$\forall x \, (A \to P(x)) \equiv A \to \forall x \, P(x)$$
$$\exists x \, (A \to P(x)) \equiv A \to \exists x \, P(x)$$

<u>Negation:</u> $\neg \forall x \, \exists y \, (x \cdot y = 1)$ $P(x,y)$

$\equiv \exists x \, \forall y \, (x \cdot y \neq 1)$ $\neg P(x,y)$

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ex all ravens are black

① U.D. to be ravens $B(b)$: "b is black"

all ravens are black : $\forall r \; B(r)$

② U.D. is all birds $B(b)$: "b is black"
$R(v)$: "v is a raven"

$$\forall t \, (R(t) \to B(t))$$

what happens to $\neg \forall t \, (R(t) \to B(t))$
$\equiv \exists t \; \neg(R(t) \to B(t))$
$\equiv \exists t \; \neg(\neg B(t) \vee B(t))$

$\equiv \exists t\,(R(t) \wedge \neg B(t))$

1.6 Rules of Inference (useful tautologies)

Mathematical argument:

$$[(S_1 \wedge S_2 \wedge S_3 \wedge \ldots \wedge S_n) \rightarrow C]$$

p

or

premises

S_1

S_2

$\vdots$

S_n

$\therefore C$ — conclusion

argument form

Valid : tautology

(ex) $[(p \rightarrow q) \wedge p] \rightarrow q$

$p \rightarrow q$

p

$\therefore q$

Valid?

$p \rightarrow q$

$\neg q$

$\therefore \neg p$