Math 321

1.5 Rules of Inference + logical equiv.

Proof ← show to true : Mathematical argument.

Statement I don't know to be true, but I think it is..

Conjecture until they are proved (or disproved)

they are an open problem.

ex If I'm a college president, then I'm not happy

$\equiv \neg(\text{college pres}) \vee \neg(\text{happy})$

$\equiv \neg(\text{college pres} \wedge \text{happy})$

How to "prove" some given conjecture?

ex $\sqrt{2}$ is irrational

1. read (know) what is being said?
2. plan an argument → proof?
3. Make the argument
4. check

2

$\sqrt{\ }$

rational $\frac{a}{b}$ ① a, b are int ② $b \neq 0$ ③ no common factors

types to know

(1) Implication $\bigcirc \to \triangle$

a) direct proof: assume $\bigcirc$, show $\triangle$

non-direct

b) contrapositive: $\neg\triangle \to \neg\bigcirc$

c) contradiction: want: $(\bigcirc \to \triangle) \equiv T$

same as: $\neg(\bigcirc \to \triangle) \equiv F$

do this $\to$ $\boxed{\bigcirc \wedge \neg\triangle \equiv F}$

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lemma: if a^2 is even $\to$ a is even

to hard!

pf: (try direct) assume a^2 is even

means $a^2 = 2 \cdot K$, K is integer

$\sqrt{a^2} = \sqrt{2K}$

$|a| =$

$a = 2 \cdot (\text{int})$

(try contrapositive)

$\neg(a \text{ is even}) \to \neg(a^2 \text{ is even})$

$\boxed{a \text{ is odd} \to a^2 \text{ is odd}} \equiv T$

even $2(\text{int})$

odd $2(\text{int}) \pm 1$

try direct for this

assume a is odd

mean $\boxed{a = 2K + 1} \to a^2 = (2K+1)^2$

$\to a^2 = 4K^2 + 4K + 1$

$\to a^2 = 2(2K^2 + 2K) + 1$

$$\boxed{a^2 = 2(\text{int}) + 1}$$

fact: $\boxed{\sqrt{2} \text{ is irrational}}$

pf assume $\neg(\sqrt{2} \text{ is irrational}) \equiv \sqrt{2}$ is rational

① a,b ints
② $b \neq 0$
③ no common factors

so $\sqrt{2} = \frac{a}{b}$

$\Rightarrow \frac{a^2}{b^2} = 2 \rightarrow a^2 = 2b^2 \Rightarrow a^2$ is even

$\Rightarrow$ by lemma a is even so $a = 2 \cdot k$, k is an int

so $a^2 = 2b^2$ is $(2k)^2 = 2b^2$

$\rightarrow 4k^2 = 2b^2$

$\rightarrow 2k^2 = b^2$

so b^2 is even and by lemma b is even

so a,b have no common factor
and a,b are both even $\Big] \equiv F$

So $\sqrt{2}$ is rational $\equiv F$

or $\sqrt{2}$ is irrational $\equiv T$

Variations & Implies

① Cases (finite)

$(p_1 \vee p_2 \vee \cdots \vee p_k) \rightarrow c$

$\equiv \underbrace{(p_1 \rightarrow c)}_{\text{case 1}} \wedge \underbrace{(p_2 \rightarrow c)}_{\text{case 2}} \wedge \cdots \wedge \underbrace{(p_k \rightarrow c)}_{\text{case k}}$

② $\bigcirc \Longleftrightarrow \triangle$ (biconditional)

i) $\bigcirc \equiv S_1 \equiv S_2 \equiv S_3 \equiv \dots \equiv \triangle$

ii) $(\bigcirc \to \triangle) \wedge (\triangle \to \bigcirc)$

Existential Proof:

find your witness

Statement: Some element has the property.

$$\exists x\, P(x)$$

① actually find the witness (constructive proof)

ex Some square is next to a cube

square 1, 4, 9, 16, 25, 36, ...

cubes 1, 8, 27, 64, ...

② Non constructive (you can't find the witness ... $\boxed{\text{but}}$ you can show it exists)