Math 321

New Toys!

Set: unordered collection of stuff (elements)

Set theory: Naive Set Theory

Notation: ① lowercase: element   uppercase: Set

if e, an element, is in a set S **use**: $e \in S$

if not $e \notin S$

② if you have a specific amount of elements

→ **Use a list**:

$$S = \{e_1, e_2, e_3, \dots, e_n\}$$

(ex) $N = \{1, 3, 5, 7, \dots, 13\}$

$S = \{2, 4, 6, 8, \dots\}$

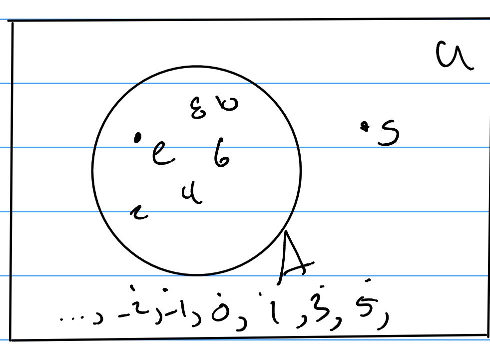
③ Set Builder notation $S = \{e \mid P(e)\}$ ← Proposition function

an element is in S if $P(e) \equiv T$

ex: $S = \{2, 4, 6, 8, 10\}$

$S = \{x \mid x \text{ is an integer} \land x = 2 \cdot k \land 2 \le x \le 10 \land k \text{ is an integer}\}$

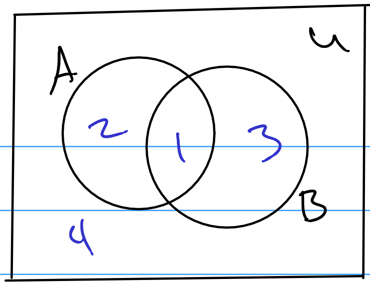
④ Venn Diagram



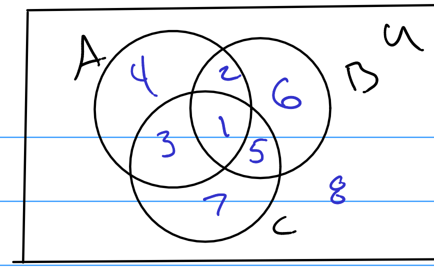
$e \in A$

$s \notin A$

2 Set



3 Sets



Membership Table

	A	B
r_1	1	1
r_2	1	0
r_3	0	1
r_4	0	0

A	B	C
1	1	1
1	1	0
1	0	1
1	0	0
0	1	1
0	1	0
0	0	1
0	0	0

Sets we should all know... **Numbers!**

① positive integers $\mathbb{Z}^+ = \{1, 2, 3, 4, \ldots\}$

② natural numbers $\mathbb{N} = \{0, 1, 2, 3, 4, \ldots\}$
(non-negative integers)

③ integers $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

④ rationals $\mathbb{Q} = \{\frac{a}{b} \mid a \in \mathbb{Z} \wedge b \in \mathbb{Z} \wedge b \neq 0$
$\wedge$ no common factors $\}$

⑤ reals $\mathbb{R} = \{d \mid d$ can be written as a **decimals**$\}$

ex $0.101001000 1 \ldots \in \mathbb{R}$

$2.3 \in \mathbb{R}$

$123 \in \mathbb{R}$

But $\mathbb{R}$ are $\mathbb{Q}$ and irrationals

$\neq 0.4aaaa\ldots$

→ **decimal:** terminates ex $\frac{1}{2} = 0.5000\ldots$
or repeats ex $0.121212 12\ldots$

$123 = 122.\overline{9}$

So always exclude $\overline{9}$ decimals.

⑥ Complex numbers $\mathbb{C} = \{a+bi \mid a, b \in \mathbb{R} \wedge i = \sqrt{-1}\}$

Basic Sets: $\phi = \{\ \}$

U : universal set

Operations:

1st "Same?" "Comparison?"

① Equality: $A = B$ means $\forall e\,(e \in A \leftrightarrow e \in B)$

Note: (Ex) $A = \{\square, \square, \smile, \frown\}$ $\square \in A$ $\smile \in A$ $\frown \in A$

$B = \{\square, \smile, \smile, \smile, \frown\}$ $\square \in B$ $\smile \in B$ $\frown \in B$

So $A = B$

b/c of this example we normally only write uniq. elements,

Y (ex) $A = \{\square, \smile, \frown\}$

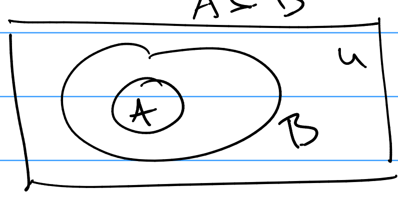
$B = \{\square, \smile, \frown\}$

② **Subset:** $A \subseteq B$ $\forall e\,(e \in A \rightarrow e \in B)$

③ **Proper Subset** $A \subset B$ $\forall e\,(e \in A \rightarrow e \in B)$

and $\exists s\,(s \notin A \wedge s \in B)$

$A \subseteq B$



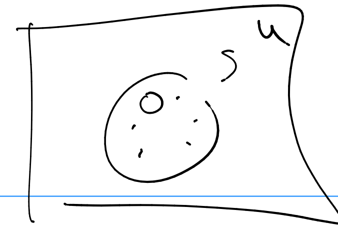
$A \subset B$

Thm

for all sets S

① $\emptyset \subseteq S$

$\forall e\,(\overbrace{e\in\emptyset}^{F} \to e\in S)$



② $S \subseteq S$

$\forall e\,(e\in S \to e\in S)$

④ "Size" Cardinality $|S|$ = # of uniq. elements in S.

Note: a) $|S| = n$ if $n \in \{0,1,2,3,\dots\}$

we would then call S finite

b) what about $|\{\dots,-2,-1,0,1,2,\dots\}|$ is not finite.

any $|S|$ that is not finite is called infinite.

$|\{2,4,6,8,\dots\}|$ infinite

$\to |\{1,2,3,4,\dots\}|$ infinite