Math 321

$\boxed{Q\text{'s}}$ 2.1 3a $A = \{ f \mid f \text{ goes from NY to ND} \}$

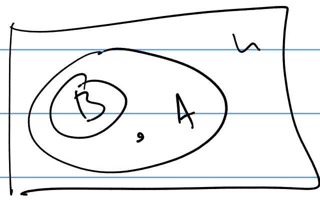
$B = \{ f \mid f \text{ goes from NY to ND non-stop} \}$

?. $A \subseteq B$ $\forall e (e \in A \to e \in B)$

?. $B \subseteq A$ $\forall e (e \in B \to e \in A)$

?. neither?

$B \subseteq A$ and $\underbrace{A \not\subseteq B}$



$\boxed{B \subset A}$

$\boxed{Q}$ $A \subseteq B \wedge B \subseteq A$

$(e \in A \to e \in B) \wedge (e \in B \to e \in A) \equiv \underline{\underline{e \in A \leftrightarrow e \in B}}$

$A = B$

Notation: ① x is a single element

② $\{x\}$ is a set that happens to have one element in it. And that element is x.

$\cancel{x = \{x\}}$ $x \in \{x\}$

ex $A = \{ \emptyset, \{a,b,c\}, \smile, \square \}$

$|A| = 4$ elements $\xrightarrow{\text{they are}}$ ① $\emptyset$

② $\{a,b,c\}$

③ $\smile$

④ $\square$

$\smile \in A$ (true) $a \in A$ (false)

$\frown \in A$ (false) $\{a,b,c\} \in A$ (true)

Ex $A = \{1, 2, 3\}$

Facts: $1 \in A$, $4 \notin A$

(Subsets) $\{1,2\} \subseteq \{1,2,3\}$ $\{2,3,4\} \not\subseteq \{1,2,3\}$

Comparison

$\emptyset$? $\{3\} \subseteq \{1,2,3\}$

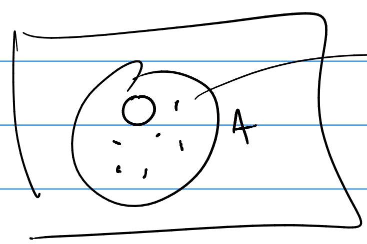
Ex $A = \{a, b, c, d, \emptyset\} \to$ A's elements

1. a
2. b
3. c
4. d
5. $\emptyset$

Facts: $\emptyset \subseteq A$ $\emptyset \in A$

$\boxed{e \in \emptyset} \to e \in A$

$F \to e \in A$



$\emptyset \subset A$

$\emptyset \subseteq \emptyset$

Ex $A = \{a, b, c\}$ $B = \{a, b, c, \{a, b, c\}\}$

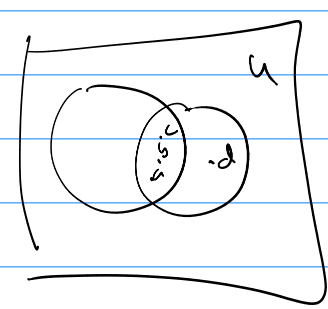
$A \subseteq B$

$A \in B$

Ex $A = \{a, b, c\}$ $B = \{a, b, c, d\}$

$A \subseteq B$

$A \in B$



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ex $A = \{a, b\}$ $\quad B = \{D, 1, \{a,b\}\}$

$A \in B$, $A \notin B$

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ex $S = \{a, b, \phi\}$ elements ① a ② b ③ $\boxed{\phi}$

$\boxed{\phi} \subseteq S$ $\qquad \phi \in S$

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ex $S = \{\{\phi\}, \{\{\phi\}\}\}$ $\qquad \phi \subseteq S$

$\phi \notin S$

Operations

① Powerset : collect all possible subsets

ex $A = \{a, b, c\}$ Subsets: take 0 → ϕ

take 0	ϕ	000
take 1	$\{a\}, \{b\}, \{c\}$	100, 010, 001
take 2	$\{a,b\}, \{a,c\}, \{b,c\}$	110, 101, 011
take 3	$\{a,b,c\}$	111

$P(A) = \{\phi, \{a\}, \{b\}, \{c\},$
$\{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$

Note: $|P(S)| = 2^{|S|}$

Prove: ex 2.1 #25 $\boxed{P(A) \subseteq P(B)}$ iff $\boxed{A \subseteq B}$

$S \in P(A) \rightarrow S \in P(B)$

idea: $A = \{e_1, e_2, \dots, e_n\}$

$\{e_1\}, \{e_2\}, \dots \{e_n\}$

so $\{e_i\} \subseteq A \rightarrow \{e_i\} \subseteq B$

$e_i \in A \rightarrow e_i \in B$

pf

Case 1 $\boxed{P(A) \subseteq P(B)} \rightarrow A \subseteq B$

Case 2 $\boxed{A \subseteq B} \rightarrow P(A) \subseteq P(B)$

assume $\boxed{e \in A \rightarrow e \in B}$

show: if $S \subseteq A \rightarrow S \subseteq B$

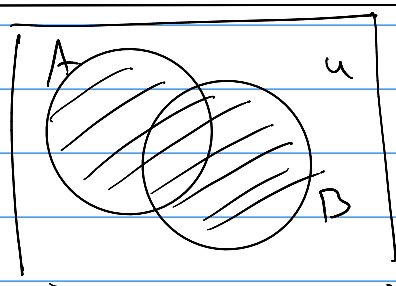
Ops

Cross product given $A_1, A_2, \dots, A_n$ sets

$$A_1 \times A_2 \times A_3 \times \dots \times A_n = \{ \underbrace{(a_1, a_2, \dots, a_n)}_{n\text{-tuple}} \mid a_1 \in A_1 \land a_2 \in A_2 \land \dots a_n \in A_n \}$$

$$|A_1 \times A_2 \times \dots \times A_n| = |A_1| \, |A_2| \dots |A_n|$$

Closed Operators

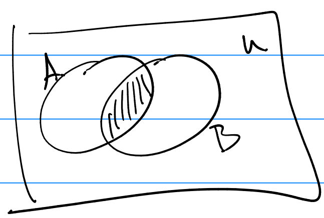


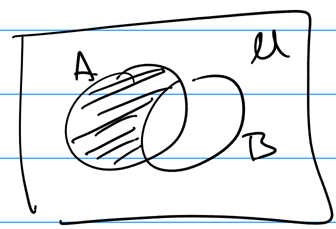
$A \cup B$

① $A \cup B = \{ e \mid e \in A \lor e \in B \}$

② $A \cap B = \{ e \mid e \in A \land e \in B \}$

$A \cap B$





$A - B$

③ $A - B = \{ e \mid e \in A \land e \notin B \}$

④ $\overline{A} = \{ e \mid e \notin A \}$

we can use logic

$$\overline{A\cup B} = \{e \mid e \notin (A\cup B)\}$$
$$= \{e \mid \neg(e\in(A\cup B))\}$$
$$= \{e \mid \neg(\boxed{e\in A} \vee \boxed{e\in B})\}$$
$$= \{e \mid \neg(e\in A)\wedge \neg(e\in B)\}$$
$$= \{e \mid e\notin A \wedge e\notin B\}$$

So $\boxed{\overline{A\cup B} = \overline{A}\cap\overline{B}}$

$$\overline{A\cap B} = \overline{A}\cup\overline{B}$$

Set Identities

$A\cap U = A$	$A\cup\emptyset = A$	Identity laws
$A\cup U = U$	$A\cap\emptyset = \emptyset$	
$A\cup A = A$	$A\cap A = A$	
$A\cup B = B\cup A$	$A\cap B = B\cap A$	

etc
}

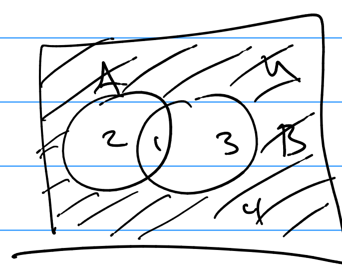
We can show: left = right

by ① logic + set builder notation

or ② Membership tables (useful for Venn Diagrams)

ex $\overline{A\cup B} = \overline{A}\cap\overline{B}$

A	B	$A\cup B$	$\overline{A\cup B}$	$\overline{A}$	$\overline{B}$	$\overline{A}\cap\overline{B}$
1	1	1	0	0	0	0
1	0	1	0	0	1	0
0	1	1	0	1	0	0
0	0	0	1	1	1	1



have same regions

2.3 Functions:

$$A \times B = \{(a,b) \mid a \in A \wedge b \in B\}$$

relationships from A to B

$$R: A \to B$$

domain — codomain

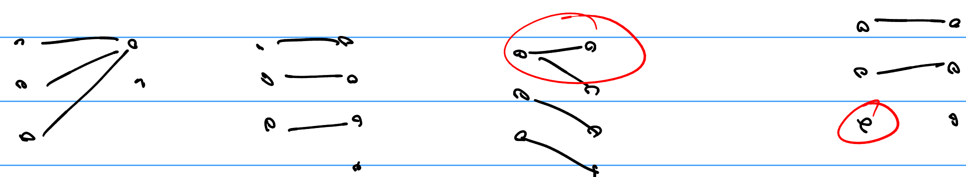
ex) A: students (Domain)

B: chairs (codomain)

range = chairs sat in

Function: restrict the relationship from $A \to B$ so that every element of A maps to B and can only go to one element of B.

ex) Arrow diagram



function — function — not a function — not a function