Math 321

Q's show $Set_1 = Set_2$

technique #1 Set builder notation and logic on Prop. function.

$Set_1 = \{e \mid P(e)\} = \{e \mid \text{logically}\} = \cdots = Set_2$

equiv. function

technique #2 b/c $A = B$ is $\forall e\,(e \in A \leftrightarrow e \in B)$

$= \forall e\,(\boxed{(e \in A \to e \in B)} \wedge (e \in B \to e \in A))$

$\equiv (A \subseteq B) \wedge (B \subseteq A)$

technique #3 Membership table

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ex $A \cap (A \cup B) = A$

technique #2 — $\boxed{\text{case 1}}$ $A \cap (A \cup B) \subseteq A$

$\boxed{\text{case 2}}$ $A \subseteq A \cap (A \cup B)$

$\boxed{\text{case 1}}$ show $A \cap (A \cup B) \subseteq A$ is if $e \in A \cap (A \cup B)$ then $e \in A$

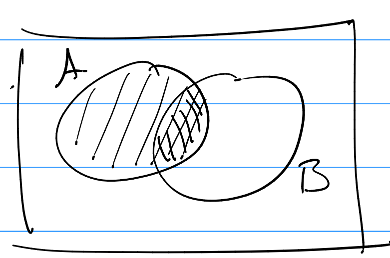
Direct Proof: assume $e \in A \cap (A \cup B)$

$\equiv e \in A \wedge e \in (A \cup B)$

$\equiv e \in A \wedge (e \in A \vee e \in B) \equiv e \in A$ (absorbtion law of logic)

$\cancel{\equiv (e \in A \wedge e \in A) \vee (e \in A \wedge e \in B)}$

Note: $A \cup (A \cap B)$
$= A$



2.3 5c $\quad f: A \to B$

domain

codomain

ex $\quad f: \mathbb{R} \to \mathbb{R}$ $\quad$ college Algebra $\quad f(x) = \dfrac{x+2}{x^2-4} = \dfrac{1}{x-2}, \; x \neq -2$

not a function for $\mathbb{R} \to \mathbb{R}$ $\quad$ b/c 2, -2 are not assigned.

so modify domain to a "natural" domain and have a partial function..

$f(x) = \dfrac{x+2}{x^2-4}$ is from $\mathbb{R} - \{2, -2\}$ to $\mathbb{R}$

natural domain

$\S$ 5c $\quad$ left overs after bit string is blocked into 8 bits (byte)

ex $\quad f(\boxed{01010011}01) = 01$

$f(010) = 010$

$f(\boxed{01010101}) = \lambda$ ← null string (bit string of no length)

Domain: all bit strings

Codomain: all bit strings

range: $\boxed{\lambda, 0, 1, 00, 01, 10, 11, \ldots, 1111111}$

all bit strings from length 0 to 7.

Q $\quad$ bit strings $\quad$ 00 11 01 01 $\quad$?

$123 = 1 \cdot 10^2 + 2 \cdot 10^1 + 3 \cdot 10^0$

$101 = 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0$

$\lfloor 4.\cancel{9993} \rfloor = 4$ $\quad$ $\lceil 4.99 \rceil = 5$

$\lfloor -4.9993 \rfloor = -5$ $\quad$ $\lceil -4.99 \rceil = -4$

Functions to know:

floor $\lfloor x \rfloor$ = int or lesser int

ceiling $\lceil x \rceil$ = int or greater int

round $|x|$ = nearest int, but $|1.5| = 2$

round$(1.1) = 1$ — round$(1.9) = 2$

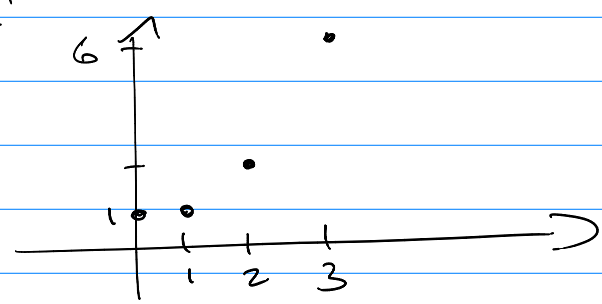
floor$(1.1) = 1$ — floor$(1.9) = 1$

ceiling$(1.1) = 2$ — ceiling$(1.9) = 2$

$n!$: #1 $0! = 1$

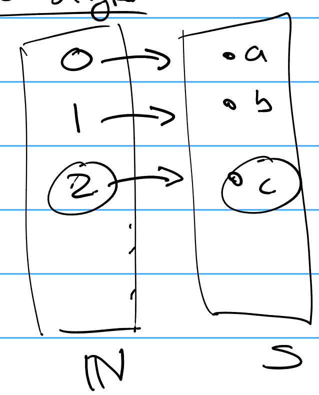
#2 $n! = n(n-1)(n-2)\cdots(1)$

$f: \mathbb{N} \to \mathbb{Z}^+$



$\boxed{7.4}$ $f: \mathbb{N} \to S$ any set

as arrow diagram



$f(0) = a$

$f(1) = b$ $\rightsquigarrow$ use the fact that $0, 1, 2, \ldots$ is wellordered

$f(2) = c$ and just give the images

$\vdots$

$\boxed{a, b, c, \ldots}$

a new way to write our function $f: \mathbb{N} \to S$

Notation: $a_0, a_1, a_2, a_3, \ldots$ is called a sequence.

ex $f(n) = 1 + 2n \quad , \quad n = 0, 1, 2, \ldots$

$\Rightarrow$ seq: $\boxed{1, 3, 5, 7, 9, \ldots} = \{1+2n\} \quad n=0,1,2,\ldots$

Sequences to $\boxed{\text{know}}$

rule		seq
$\{n\}$	$n=0,1,2,\ldots$	$0, 1, 2, \ldots$
$\{n^2\}$	$n=0,1,2,\ldots$	$0, 1, 4, 9, 16, \ldots$
$\{n^3\}$	$n=0,1,2,\ldots$	$0, 1, 8, 27, 64, \ldots$
$\{n!\}$	$n=0,1,2,\ldots$	$1, 1, 2, 6, 24, 120, \ldots$

$\Rightarrow$ given the function $\longrightarrow$ seq

given the seq $\longrightarrow$ guess? the function.

arithmetic seq: $\{a + bn\} \quad n=0,1,2,\ldots$

ex $\{2 + 3n\} \quad n=0,1,2,\ldots$

seq: $2, 5, 8, 11, \ldots$

3 3 3

ex seq: $-4, 1, 6, 11, \ldots \Rightarrow \boxed{\{-4+5n\} \quad n=0,1,2,\ldots}$

5 5 5

geometric seq $\{a r^n\} \quad n=0,1,2,\ldots$

ex $\{2 \cdot 3^n\} \quad n=0,1,2,\ldots$

seq: $2, 6, 18, 54, \ldots$ $\qquad 3(18) \quad 3(10+8)$

divide 3 3 $\times 3$ $\times 3$ $\qquad 30+24$

given a function $\{a_n = f(n)\}$ $n = 0, 1, 2, \ldots$

we say it is of closed form.

VS open form, recursive, inductive formulas..

Step 1 told the starting value(s). these are the basis

Step 2 formula to make new values from old.

ex $a_0 = 2$ formula: $a_n = 3a_{n-1}$ $n = 1, 2, 3, \ldots$

$a_0 = 2$

$a_1 = 3 \cdot 2 = 6$

$a_2 = 3 \cdot 6 = 18$

$\vdots$

$n=1$

$a(1) = 3a(0) = 3 \cdot 2 = 6$

a_{n-1}

ex Fibonacci Numbers

Basis: $f_0 = 0$, $f_1 = 1$

Formula: $f_n = f_{n-1} + f_{n-2}$ $n = 2, 3, 4, \ldots$

$f_3 = f_2 + f_1$

Seq: $0, 1, 1, 2, 3, 5, 8, \ldots$

f_0 f_1 $f_2 = f_1 + f_0$

$\boxed{\text{Sums}}$ $\rightarrow$ add up some of a seq's values...

<u>Seq:</u> $a_0, \underbrace{\boxed{a_1, a_2, a_3, a_4}}_{a_1+a_2+a_3+a_4 \,\leftarrow\, \text{Sum}}, a_5, a_6, \ldots$

<u>Notation:</u> $\displaystyle\sum_{i=n}^{m} a_i = a_n + a_{n+1} + \cdots + a_m$