Math 321

Q's 4.3 39 e) $\gcd(213, 117)$ Bezout's $3 = s \cdot 213 + t \cdot 117$

$3 = \gcd(213,117)$	$213 = 1 \cdot 117 + 96$	$3 = 12 - 9$
$= \gcd(117,96)$	$117 = 1 \cdot 96 + 21$	$3 = 12 - (21 - 12) = 2 \cdot 12 - 21$
$= \gcd(96,21)$	$96 = 4 \cdot 21 + 12$	$3 = 2(96 - 4 \cdot 21) - 21 = 2 \cdot 96 - 9 \cdot 21$
$= \gcd(21,12)$	$21 = 1 \cdot 12 + 9$	$3 = 2 \cdot 96 - 9(117 - 96) = 11 \cdot 96 - 9 \cdot 117$
$= \gcd(12,9)$	$12 = 1 \cdot 9 + \boxed{3}$	$3 = 11(213 - 117) - 9 \cdot 117$
$= \gcd(9,3) = 3$	$9 = 3 \cdot 3 + 0$	

$$\boxed{\gcd(213,117) = 3 = 11 \cdot 213 - 20 \cdot 117}$$

#43 Extended Euclidean Algorithm

$\gcd(a,b)$ $a \geq b$ r's ex $\gcd(213,117)$

	$r_0 = a$	$213 = r_0$
$\gcd(a,b)$	$r_1 = b$	$117 = r_1$
$\gcd(r_0,r_1)$	$r_0 = (q_1) r_1 + (r_2)$	$213 = \overset{=q_1}{1} \cdot 117 + 96 = r_2$
$\gcd(r_1,r_2)$	$r_1 = (q_2) r_2 + (r_3)$	$117 = \overset{=q_2}{1} \cdot 96 + 21 = r_3$
$\gcd(r_2,r_3)$	$r_2 = (q_3) r_3 + (r_4)$	$96 = \overset{=q_3}{4} \cdot 21 + 12 = r_4$
	$\vdots$	$21 = \overset{=q_4}{1} \cdot 12 + 9 = r_5$
	$r_{n-1} = \gcd$	$12 = \overset{=q_5}{1} \cdot 9 + \boxed{3} = r_6$
	$+ (0) = r_n$	$9 = \overset{=q_6}{3} \cdot 3 + 0 = r_7$

Extended Alg. add s_i, t_i to our list

$r_0 = a$	$s_0 = 1$	$t_0 = 0$	$s_k = s_{k-2} - q_{k-1} s_{k-1}$
$r_1 = b$	$s_1 = 0$	$t_1 = 1$	$t_k = t_{k-2} - q_{k-1} t_{k-1}$
$r_0 = (q_1) r_1 + (r_2)$	$s_2 =$	$t_2 =$	
$r_1 = (q_2) r_2 + (r_3)$	s_3		
$r_2 = (q_3) r_3 + (r_4)$			
$\vdots$			
$r_{n-1} = \gcd$			
$+ (0)$			

ex

$$
\begin{array}{llll}
 & 213 = r_0 & s_0 = 1 & t_0 = 0 \\
 & 117 = r_1 & s_1 = \boxed{0} & t_1 = \boxed{1} \\
213 = \boxed{1}^{q_1} \cdot 117 + 96 = r_2 & & s_2 = \boxed{1} & t_2 = \boxed{-1} \\
117 = \boxed{1}^{q_2} \cdot 96 + 21 = r_3 & & s_3 = \boxed{-1} & t_3 = \boxed{2} \\
96 = \boxed{4}^{q_3} \cdot 21 + 12 = r_4 & & s_4 = \boxed{5} & t_4 = \boxed{-9} \\
21 = \boxed{1}^{q_4} \cdot 12 + 9 = r_5 & & s_5 = \boxed{-6} & t_5 = \boxed{11} \\
12 = \boxed{1}^{q_5} \cdot 9 + \boxed{3} = r_6 & & s_6 = 11 & t_6 = -20 \leftarrow \\
9 = 3^{q_6} \cdot 3 + 0 = r_7 & & 3 = 11 \cdot 213 - 20 \cdot 117 &
\end{array}
$$

Induction: show $\forall n P(n)$ is true but $n = 0, 1, 3 \ldots$ (∞ cases) that are well ordered

forms the tautology

$$\Big[\underbrace{\underbrace{P(1^{st}\text{ case})}_{} \wedge \underbrace{\forall k \big[\big(P(1^{st}\text{ case}) \wedge P(2^{nd}\text{ case}) \wedge \ldots \wedge P(k^{th}\text{ case})\big) \to P(k+1^{st}\text{ case})\big]}_{}}_{T}\Big] \to \underbrace{\forall n P(n)}_{T}$$

So show $\boxed{\text{left is true}}$ then $\forall n P(n)$ is true.

means show $P(1^{st}\text{ case})$ is true : Basis Step

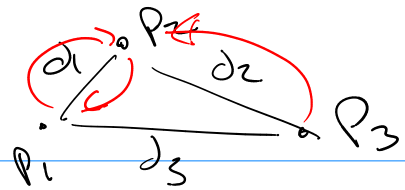
and show $\big(\underbrace{P(1^{st}) \wedge P(2^{nd}) \wedge \ldots \wedge P(k^{th})}_{}\big) \to \underbrace{P(k+1^{st})}_{}$ is true : Induction step

ex $\forall n P(n)$ $n = 3, 5, 7, 9, \ldots$

Is if an odd number of people are standing uniq. distances apart and everyone throws a snowball at the closest person then there is at least one person never hit with a snowball.

Pf: **Basis:** show $P(1^{st}\text{ case})$ is true

3 people have snowball fight



$d_1 \neq d_2 \neq d_3$

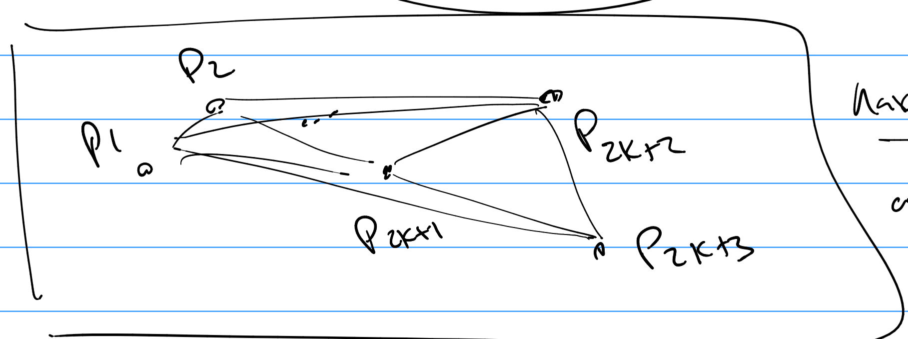
d_1 is min.

P_3 survives so $\boxed{\text{true!}}$

Inductive Step assume $\underbrace{P(1^{st})}_{\text{people}=3} \wedge \underbrace{P(2^{nd})}_{\text{people}=5} \wedge \dots \wedge \underbrace{P(k^{th})}_{\text{people}=2K+1}$ — Inductive Hypothesis

show $P(K+1^{st})$
$\uparrow$
people $= 2K+3$

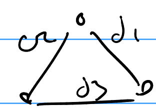
does $2K+3$ have a survivor if it worked for $\boxed{3, 5, \dots, 2K+1}$



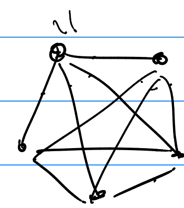
have $2K+3$ people

of these there are a closest two who always hit each other the other $2K+1$ by Inductive Hypothesis will always have one survivor.

$P_1, P_2, \dots, P_{2K+3}$



3 people



5 people

$4+3+2+1 = \frac{4(5)}{2} = 10$

Prove: $1+2+3+\dots+n = \frac{n(n+1)}{2}$ $\quad n = 1, 2, 3, \dots$

Pf **Base:** $\underbrace{P(1^{st}\text{ case})}_{n=1}$: " $1 = \frac{1(1+1)}{2}$ " $\boxed{\text{true}}$

Weak Inductive: assume $\underbrace{P(1^{st})}_{n=1} \wedge \underbrace{P(2^{nd})}_{n=2} \wedge \dots \wedge \boxed{\underbrace{P(k^{th})}_{n=k}}$

show $\boxed{\underbrace{P(k+1^{st})}_{n=k+1}}$

$P(1^{st})$: "$1 = \frac{1(1+1)}{2}$" ($n=1$)

$P(2^{nd})$: "$1+2 = \frac{2(2+1)}{2}$" ($n=2$)

$P(3^{rd})$: "$1+2+3 = \frac{3(3+1)}{2}$" ($n=3$)

$\vdots$

$P(k^{th})$: "$1+2+3+\dots+k = \frac{k(k+1)}{2}$" ($n=k$) — Inductive Hypothesis

$P(k+1^{st})$: "$1+2+3+\dots+k+(k+1) \stackrel{?}{=} \frac{(k+1)(k+2)}{2}$" ($n=k+1$)

Take: $(1+2+3+\dots+k)+(k+1) \underset{I.H.}{=} \frac{k(k+1)}{2} + \frac{(k+1)}{1} = \frac{k(k+1)+2(k+1)}{2}$

$= \frac{(k+1)(k+2)}{2}$ true

Prove: $n=2,3,\dots$ n is prime or a product of primes.

pf Basis: $P(1^{st}\text{ case})$ ($n=2$) 2 is prime so true.

Inductive: assume $n=2, n=3, n=4, \dots, n=k$ are prime or a prod. of primes.

show $n=k+1$