Math 321

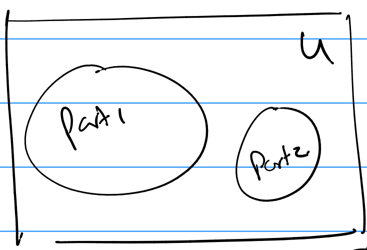
Counting

Goal: $|\text{task}|$

6.1 Basics: And vs Or

① Sum Rule   task = do part 1 or do part 2
and I know part 1, part 2 are disjoint.

how many toys can you pick from if there are 6 trucks and 10 blocks?



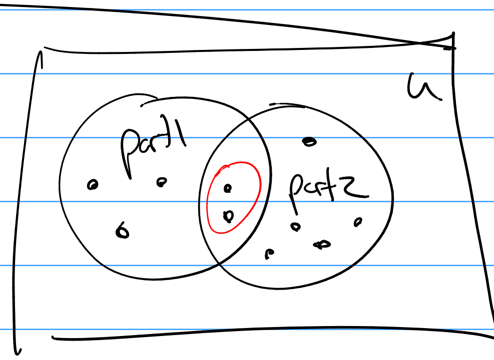
$|\text{task}| = |\text{part1}| + |\text{part2}|$

$|\text{task}| = 6 + 10 = \boxed{16}$

② what about non-disjoint sets?

$$|A \cup B| = |A| + |B| - |A \cap B|$$

Inclusion / Exclusion Principle



ex) $|\text{task}| = 5 + 7 - 2$
$= 12 - 2 = 10$

In general $|A_1 \cup A_2 \cup A_3 \cup \dots \cup A_n| = |A_1| + |A_2| + \dots + |A_n|$

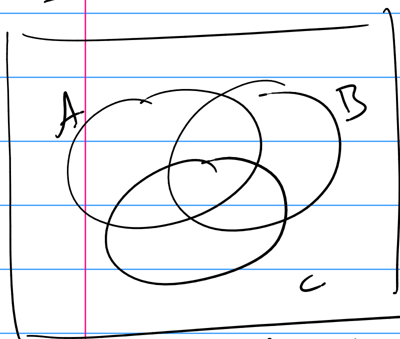
$- |A_1 \cap A_2| - |A_1 \cap A_3| - \dots - |A_{n-1} \cap A_n|$

$+ |A_1 \cap A_2 \cap A_3| + |A_1 \cap A_2 \cap A_4| + \dots$

$-$ (quads)

$\vdots$

$\pm$



$$|A \cup B \cup C| = 4 + 4 + 4 - 2 - 2 - 2 + 1 = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

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(3) **Product Rule** task = do part 1 and then part 2

$$|A_1 \times A_2| = |A_1|\,|A_2|$$

$$|\text{task}| = |\text{part 1}|\,|\text{part 2}|$$

$$\rightarrow |A_1 \times A_2 \times \dots \times A_n| = |A_1|\,|A_2| \dots |A_n|$$

ex **task:** How many ways to fill out a multiple choice ans. sheet if for problems 1 to 10 they are T/F, prob 11 to 15 are 4 options and you can select from 0 to all 4. And probs 16 to 20 are select one of A, B, C, D.

ex
1 ⊗T ○F
2 ○T ⊗F
3
⋮
10 ○T ⊗F
11 ○ ○ ○ ○
15 ○ ○ ○ ○
16 A B C D
20

$$|\text{task}| = \underset{2}{|\text{prob 1}|} \cdot \underset{2}{|\text{prob 2}|} \cdot \dots \cdot |\text{prob 20}|$$

$$|\text{task}| = \underbrace{2 \cdot 2 \cdots 2}_{10\text{ probs}} \cdot \underbrace{16 \cdots 16}_{\substack{11\text{ to }15\\ 5\text{ probs}}} \cdot \underbrace{4 \cdots 4}_{5\text{ probs}} = \boxed{2^{10}\, 16^5\, 4^5}$$

blank or mark 1 or mark 2 or mark 3 or all

1 + 4 + 6 + 4 + 1

Note: using Sum Rule to not count a problem. (disjoint)

$$|\text{task}| = |\text{part 1}| + |\text{part 2}| + \dots + |\text{part } n|$$

ex (basketball)

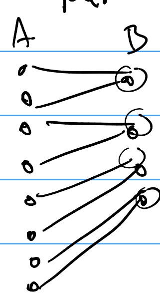
$$|\text{every team}| = \left|\begin{matrix}\text{teams} \\ \text{of 0 girls}\end{matrix}\right| + \underbrace{\left|\,\left|\begin{matrix}\text{teams of} \\ \text{exactly 1 girl}\end{matrix}\right| + \dots + \left|\begin{matrix}\text{teams of} \\ \text{5 girls}\end{matrix}\right|\,\right|}_{\text{teams of at least one girl}}$$

$$\left|\begin{matrix}\text{teams of} \\ \text{at least one girl}\end{matrix}\right| = |\text{every team}| - |\text{no girls}|$$

4 Division Rule

procedure has n ways to do it. But every way, w, has exactly d of the n-ways correspond to w.

Functional Mindset



$|A| = 8$ for each pairing way there are 2 corresponding.

$|B| = \frac{8}{2} = 4$

Note: $0! = 1$

$4 \cdot 3 \cdot 2 \cdot 1 = 4!$

table example: 4 people $P_1\ P_2\ P_3\ P_4$

1. $|\text{sit in a straight line}| = 4!$
2. Said $P_1 P_2 P_3 P_4 = P_4 P_3 P_2 P_1$ $\quad \overrightarrow{P_1 P_2 P_3 P_4}$, $\overleftarrow{P_1 P_2 P_3 P_4}$

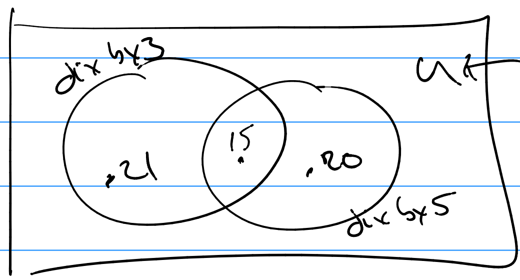
$$|\text{task}| = \frac{4!}{2} = \frac{4 \cdot 3 \cdot \cancel{2} \cdot 1}{\cancel{2}} = \boxed{12}$$

(3) Sit in circle

$$\frac{4!}{4 \cdot 2} = \boxed{3}$$

(ex) how many numbers from $\overbrace{13 \text{ to } 11021}^{\text{including } 13,\ 11021}$ are div. by
<u>3 and 5?</u> <u>3 or 5?</u>

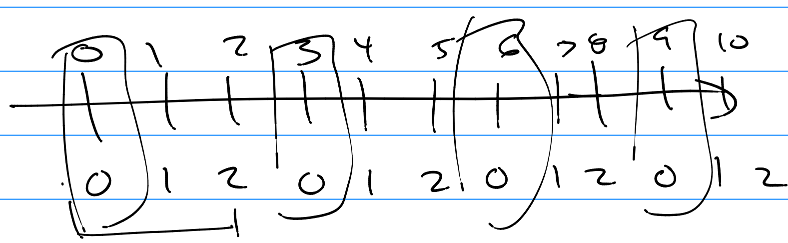
fact #1



U = all numbers of interest

$$|3 \text{ or } 5| = |3| + |5| - |\underbrace{3 \text{ and } 5}_{\text{div by } 15}|$$

fact #2



div by 3
(mod 3) = 0

so div by 3 $\approx \frac{1}{3}$ of numbers
div by 5 $\approx \frac{1}{5}$ of numbers
div by 15 $\approx \frac{1}{15}$ of numbers

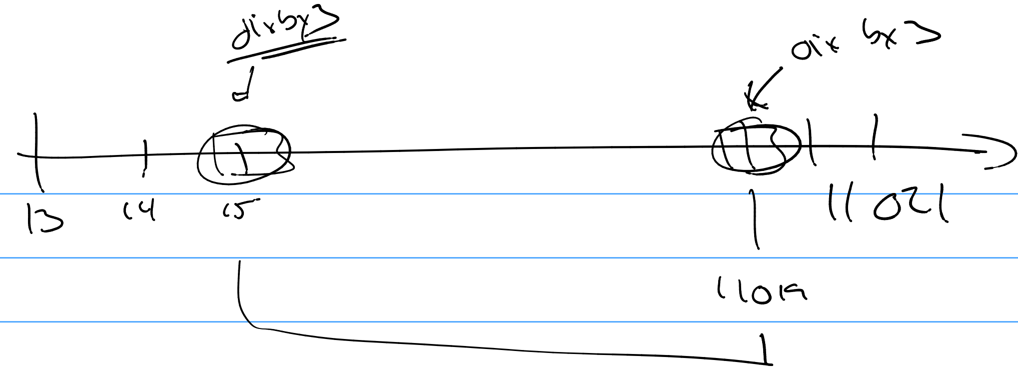
$$|\text{numbers}| = 11021 - 13 + 1 = \boxed{11009}$$

15 + 0·3
15 + 1·3 = 18
15 + 2·3 = 21
15 + (3668)·3 =

$$|3| \approx \frac{11009}{3} = 3669.\square \to \begin{matrix} 3669 \\ 3670 \end{matrix}$$

$$|5| \approx \frac{11009}{5} = \_\_\_\_$$

$$|15| \approx \frac{11009}{15} = \_\_\_\_$$



ex bit strings of length 7 that either begin with 00 or end with 111?

$$|\text{task}| = \left|\begin{matrix}\text{begin with}\\ 00\end{matrix}\right| + \left|\begin{matrix}\text{end with}\\ 111\end{matrix}\right| - \left|\begin{matrix}\text{begin } 00\\ \text{end } 111\end{matrix}\right|$$

Start with 00 and fill rest 5

$$= (1 \cdot 2^5) + (2^4 \cdot 1) - (1 \cdot 2^2 \cdot 1)$$

00 $b_1 b_2$ 111

00|00000

00|00001

00|10001

$$= \boxed{2^5 + 2^4 - 2^2}$$