Math 321

$\boxed{Q5}$ 6.1 #31 |plates| = 2 or 3 uppercase letters followed by 2 or 3 digits.

|task| ✗ how to make the plates?

Experiment
AAA 22
AB 315
⋮

tech #1)
Select 2 letters and 2 digits $= 26^2 \cdot 10^2$
or Select 2 letters and 3 digits $= 26^2 \cdot 10^3$
or Select 3 letters and 2 digits $= 26^3 \cdot 10^2$
or Select 3 and 3 $= 26^3 \cdot 10^3$ +

$$\boxed{26^2 10^2 + 26^2 10^3 + 26^3 \cdot 10^2 + 26^3 \cdot 10^3}$$

$$= 26^2 10^2 (1 + 10 + 26 + 260) = ??$$

tech #2
$$\boxed{(\underbrace{26^2}_{\text{take 2 letters}} \overset{\text{or}}{+} \underbrace{26^3}_{\text{3 letters}}) \overset{\text{and}}{} (\underbrace{10^2}_{\text{take 2 digit}} \overset{\text{or}}{+} \underbrace{10^3}_{\text{3 digits}})} = 26^2 10^2 + 26^2 10^3 + 26^3 10^2 + 26^3 10^3$$

tech #3
$$26^2 \cdot 10^2 \cdot (1 + 10 + 26 + 26 \cdot 10)$$

AA11
AA112
D AA11

Pigeonhole Principle

$\boxed{\text{Th}^{\text{m}}}$ $K \in \mathbb{Z}^+$: If $K+1$, or more objects, placed into K boxes, then at least one box has at least 2 objects.

K boxes: [|||] [||] [|||] ... [||]
1 2 3 ... K

→ at least 1 from (1 to K) = N
→ at least (2) from (K+1 to 2K) = N
→ at least (3) from (2K+1 to 3K) = N

$\boxed{\text{Th}^{\text{m}}}$ N objects placed into K boxes, then at least one box has at least $\left\lceil \frac{N}{K} \right\rceil$ objects

ex: pick blind socks. How many needed to get 2 pairs of same color if you have grey, red, blue, black, and white socks?

task is get 4 socks (objects) of same color (boxes).

$\boxed{\text{P.H.P}}$ at least one ~~box~~ color with $\left\lceil \frac{N}{K} \right\rceil$ ~~objects~~ socks

find this = # of socks (N)

K = 5

Find N such that $\left\lceil \frac{N}{5} \right\rceil = 4$

$\boxed{N = 16}$ b/c $\left\lceil \frac{16}{5} \right\rceil = \left\lceil 3 + \frac{1}{5} \right\rceil = 4$

Note: How solve $N = ?$ $\left\lceil \frac{N}{K} \right\rceil = p$

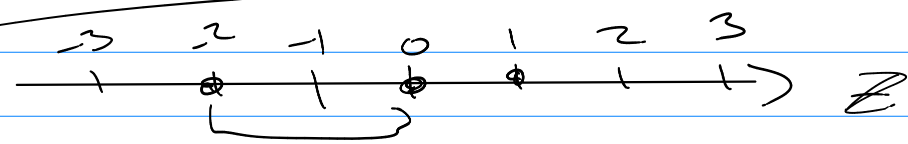
$N = (K-1)p + 1$

Not p.h.p. but look like it --

ex **Socks:** in drawer $|grey| = 5$ $|black| = 3$
$|white| = 7$ $|blue| = 9$
$|red| = 1$

how many socks to get if you need 2 white pairs?
not p.h.p. b/c it asks for specific color.

worst case: $\underbrace{5+3+9+1}_{\text{get all socks but white}} + 4$ — 2 white pair $= \boxed{22}$

ex


Q: how many points can you pick before you **must** have a mid point on integer values?

objects = points **box = ???**

but it is asking about mid points — $p_1, p_2 \to M = \frac{p_1+p_2}{2}$

Goal M is supposed to be an integer $M = \frac{p_1+p_2}{2}$
says $p_1 + p_2 = $ even

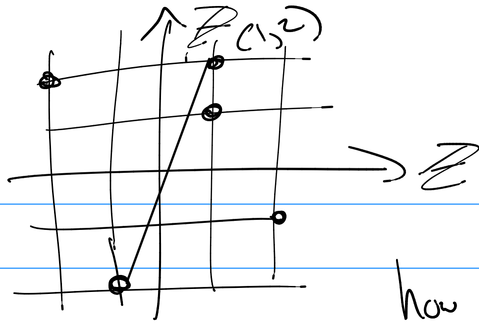
box = parity → even or odd

$|box| = 2$

Now: shared parity = integer mid point

So pick 3 points (more than 2) gives **int. M.**

2D



$p_1 = (x_1, y_1)$

$p_2 = (x_2, y_2)$

$$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

how many points so a midpoint has integer values?

objects = points

boxes = shared parity (odd, odd), (odd, even), (even, odd), (even, even) $\rightarrow$ |boxes| = 4

by P.H.P $\rightarrow$ pick 5 points

6.3 $\quad P(n,r) = \dfrac{n!}{(n-r)!} \qquad C(n,r) = \dbinom{n}{r} = \dfrac{n!}{r!\,(n-r)!}$

<u>Note:</u> $0! = 1$

$n \geq 1 \qquad n! = n(n-1)(n-2)\cdots(2)(1)$

$P(n,r)$ permutation: pick r from n and order matters

$C(n,r)$ combination: choose r from n and order does not matter.

<u>Permutation</u>

pick r from n and order matters

|task| $\rightarrow$ order all n people $\quad n(n-1)\cdots(1) = n!$

$$\underbrace{p_1\, p_2\, p_3\, p_4}_{r\text{-people}} \;\; \underbrace{\cdot \;\; p_n}_{(n-r)\text{ people}}$$

$(n-r)!$ overcount of above

so $P(n,r) = \dfrac{n!}{(n-r)!}$

(ex) pick 5 people from 21 for the b-ball game.

$$P(21,5) = \frac{21!}{16!} = 21 \cdot 20 \cdot 19 \cdot 18 \cdot 17$$

Combinations

choose r from n (order does not matter)

$|\text{task}| \to$ arrange all n people $= n!$

$$\underbrace{p_1\, p_2 \cdots p_r}_{\text{chosen}} \Big| \underbrace{p_{r+1} \cdots p_n}_{\text{not been chosen}}$$

$r!$ arrangements

$(n-r)!$ arrangements

$$\boxed{C(n,r) = \frac{n!}{r!\,(n-r)!}}$$

(ex) choose 6 people for a group from 32 students.

$$C(32,6) = \frac{32!}{6!\;26!}$$

Application: task: you have n students plus me (Mark)
(so $n+1$ people)
$\to$ choose k people for a grading committee.

$$|\text{task}| = C(n+1,k) = \binom{n+1}{k} = \frac{(n+1)!}{k!\,(n+1-k)!}$$

VS Someone wants to know $\left|\begin{matrix}\text{Mark on}\\ \text{Committee}\end{matrix}\right| \overset{+}{\text{or}} \left|\begin{matrix}\text{Mark } \underline{\underline{\text{not}}}\\ \text{on comm}\end{matrix}\right|$

$$\left|\begin{matrix}\text{Mark on}\\ \text{committee}\end{matrix}\right| = \underbrace{\binom{1}{1}}_{\substack{\text{take}\\ \text{Mark}}} \underbrace{\cdot}_{\underline{\text{and}}} \underbrace{\binom{n}{k-1}}_{\substack{\text{choose other}\\ k-1 \text{ spots}\\ \text{from students}}}$$

$$\left|\begin{matrix}\text{Mark } \underline{\underline{\text{not}}}\\ \text{on committee}\end{matrix}\right| = \underbrace{\binom{n}{k}}_{\substack{\text{choose } k\\ \text{from only students.}}}$$

So
$$\boxed{\underbrace{\binom{n+1}{k}}_{\underline{\underline{\text{all com.}}}} = \underbrace{\binom{n}{k-1}}_{\left|\text{with Mark}\right|} \underbrace{+}_{\text{or}} \underbrace{\binom{n}{k}}_{\left|\text{without Mark}\right|}}$$

Combinatorial Proofs:

$$|\text{task}| = \text{formula } \#1$$

Same task

$$|\text{task}| = \text{formula } \#2$$

means

$$\boxed{\text{formula } \#1 = \text{formula } \#2}$$

ex from n math profs, n C.S. profs
choose n people for a committee
and the chairperson is from mathematics

$$\underbrace{\underbrace{\text{math}}_{n}\ \underbrace{\text{CS}}_{n}}_{2n \text{ people}}$$

(show: $\displaystyle\sum_{k=1}^{n} k\binom{n}{k}^2 = n\binom{2n-1}{n-1}$)

$$1\cdot\binom{n}{1}\binom{n}{1} + 2\cdot\binom{n}{2}\binom{n}{2} + \cdots + n\binom{n}{n}\binom{n}{n}$$