Math 321

Q's 6.3 #21 $\quad S = \{A, B, C, D, E, F, G\} \quad |S| = 7$

permutate these letters
arrange

$$|\text{all arrangements}| = 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1 = 7!$$

$$= P(7,7) = \frac{7!}{0!} = 7!$$

a) have BCD in permutation.

tech #1
Step 1 get BCD (1 way to do this)
Step 2 place $\boxed{BCD}$ (5 ways)
step get rest $4!$
$\rightarrow 1\cdot 5\cdot 4! = \boxed{5!}$

tech #2 $\quad \{A, B, C, D, E, F, G\}$ "old" set

You say we need BCD $\rightarrow$ Set $= \{BCD, A, E, F, G\}$

arrange 5 things $\boxed{5!}$

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26 c $\quad$ 16 people $=$ 3 girls $+$ 13 guys

(a) choose 10 $\quad \binom{16}{10} = \frac{16!}{10!\,6!} = \frac{\cancel{16}\cdot\cancel{15}\cdot 14\cdot 13\cdot\cancel{12}\cdot 11}{\cancel{6}\cdot\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}}$

order does not matter

$$= 14\cdot 13\cdot 11\cdot 4 = ?$$

(c) Heart of prob: $\quad |\text{all teams}| = |\text{no girls}| + |\text{exactly one}| + |\text{exactly 2}| + |\text{exactly 3}|$

4 "all guys"

$$\binom{16}{10} = \binom{13}{10} + \binom{3}{1}\cdot\binom{13}{9} + \binom{3}{2}\cdot\binom{13}{8} + \binom{3}{3}\cdot\binom{13}{7}$$

$$\left|\text{at least 1 girl}\right| = \left|\begin{matrix}\text{exactly}\\1\end{matrix}\right| + \left|\begin{matrix}\text{exactly}\\2\end{matrix}\right| + \left|\begin{matrix}\text{exactly}\\3\end{matrix}\right|$$

tech #1 $\binom{3}{1}\binom{13}{9} + \binom{3}{2}\binom{13}{8} + \binom{3}{3}\binom{13}{7}$

$$= \frac{3!}{1!\,2!}\,\frac{13!}{7!\,4!} + \frac{3!}{2!\,1!}\,\frac{13!}{8!\,5!} + \frac{3!}{3!\,0!}\,\frac{13!}{7!\,6!} = ?$$

tech #2 $\left|\text{all}\right| - \left|\begin{matrix}\text{no}\\\text{girls}\end{matrix}\right| = \left|\text{at least 1 girl}\right|$

$$\binom{16}{10} - \binom{13}{10}$$

$$= \frac{16!}{10!\,6!} - \frac{13!}{10!\,3!} = ?$$

$\boxed{\text{Ch 8}}$ recurrence relations

Basis Step: $a_0 = 1$

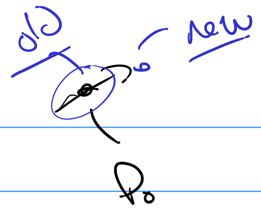
Inductive Step: $a_n = 2a_{n-1}$ ← recurrence relation

Use:

$$\left.\begin{aligned} a_0 &= 1 \\ a_1 &= 2a_0 = 2(1) = 2 \\ a_2 &= 2a_1 = 2(2) = 4 \\ &\vdots \end{aligned}\right\} \rightarrow \text{Seq: } 1, 2, 4, 8, 16, 32, \ldots$$

Modeling: why recurrence relations?

b/c lots of problems have new stuff = function (old stuff)



f', f''

Doubling example

$\boxed{ex}$ ① given one grain of rice on day 0. $a_0 = 1$

② each day after you get twice day before. $a_n = 2a_{n-1}$

Seq $\underset{0}{1}, \underset{1}{2}, \underset{2}{4}, \underset{3}{8}, \underset{4}{16}, \underset{5}{32}, \ldots$

$\boxed{ex}$ 3 boys go out to get all the cookies in town. get back home and will share tomorrow. $\boxed{\text{But}}$ boy 1 doesn't want to wait so he divides pile into 3, one cookie is left over (and he gives it to dog). hides his pile. Boy 2 does the same to leftovers a bit later. Boy 3 does the same to leftovers a bit later.

Next day they just share leftovers and no cookie for dog.

$\boxed{Q:}$ how many cookies @ start? how many for each kid? how many for dog = 3.

New vs old piles? a_0 = 1st pile

boy 1 $\to$ $a_0 = \underbrace{a_1}_{me} + \underbrace{a_1 + a_1}_{left} + \underbrace{1}_{dog}$

$a_0 = 3a_1 + 1$

boy 2 $\to$ $a_1 = a_2 + a_2 + a_2 + 1$

$a_1 = 3a_2 + 1$

boy 3 $\to$ $a_2 = a_3 + a_3 + a_3 + 1$

$a_2 = 3a_3 + 1$

wrong!??

Next Morning: a_0 $\boxed{\text{know } 3 \mid a_0}$ fact

Day 3 $a_1 = 3\left(\frac{1}{2}a_0\right) + 1$

$a_1 = \frac{3}{2}a_0 + 1$

Day 2 $a_2 = \frac{3}{2}a_1 + 1$

Day 1 $a_3 = \frac{3}{2}a_2 + 1$

Basis: $a_0 = ?$ $3 \mid a_0$

Induction: $\boxed{a_n = \frac{3}{2}a_{n-1} + 1}$

$a_0 = 3, 6, 9, 12, \ldots$

$a_1 = 1\square$

$a_2 = 1\square$

$\boxed{a_3 \leftarrow \text{find.}}$

Finding closed formulas from rec. relations

#1 guess and check

ex $\boxed{a_0 = 1}$ $a_1 = 2$ $a_2 = 4$ $a_3 = 8$ $a_4 = 16$ ...

$(a_n = 2\,a_{n-1})$ ← rec. relation.

guess $a_n = 2^n$

check: (1) Basis $a_0 = 2^0 = 1$ yes!

(2) Induction: $a_n = 2^n$ $a_{n-1} = 2^{(n-1)}$

$a_n \overset{?}{=} 2\,a_{n-1}$

$2^n \overset{?}{=} 2^1 \cdot 2^{n-1}$

$2^n \overset{?}{=} 2^n$ yes!

tech #2 iteration (forward or backward)

ex $a_n = \frac{3}{2} a_{n-1} + 1$

Forward:

a_0

$a_1 = \frac{3}{2} a_0 + 1$

$a_2 = \frac{3}{2} a_1 + 1 = \frac{3}{2}\left(\frac{3}{2} a_0 + 1\right) + 1 = \left(\frac{3}{2}\right)^2 a_0 + \left(\frac{3}{2}\right) + 1$

$a_3 = \frac{3}{2} a_2 + 1 = \frac{3}{2}\left(\left(\frac{3}{2}\right)^2 a_0 + \frac{3}{2} + 1\right) + 1$

$\Rightarrow a_3 = \left(\frac{3}{2}\right)^3 a_0 + \left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right)^1 + \left(\frac{3}{2}\right)^0$

$$a_k = \left(\frac{3}{2}\right)^k a_0 + \left[\left(\frac{3}{2}\right)^{k-1} + \dots + \left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right)^1 + \left(\frac{3}{2}\right)^0\right]$$

(like $\sum_{j=0}^{n} ar^j = a\left[\frac{r^{n+1} - 1}{r - 1}\right]$

$$\boxed{a_k = \left(\frac{3}{2}\right)^k a_0 + \frac{\left(\frac{3}{2}\right)^k - 1}{\left(\frac{3}{2}\right) - 1}}$$

backward $a_k = \frac{3}{2} a_{k-1} + 1$

$a_{k-1} = \frac{3}{2} a_{k-2} + 1$

$a_k = \frac{3}{2}\left(\frac{3}{2} a_{k-2} + 1\right) + 1 = \left(\frac{3}{2}\right)^2 a_{k-2} + \frac{3}{2} + 1$

Ex tile tile

task put down n-tiles but no in pattern.

a_n = # of ways to put down the tiles.

task (put down n-tiles with no) = a_n

a_n = (place) and (other $n-1$ have no)

$1 \cdot a_{n-1}$

+ or

(place) and (other $n-2$ have no)

$1 \cdot a_{n-2}$

$$\boxed{a_n = a_{n-1} + a_{n-2}}$$ recurrence relation

Basis: a_1 ← tiling of length 1

a_2 ← tiling of length 2

$a_0 = 1$

$a_1 = 2$

$a_2 = 3$

$a_3 = 5$, $a_4 = 8$, $a_5 = 13$, ..