Math 321

Q's

6.4 #19 $\quad \binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}$

Algebra?

$$\boxed{\binom{a}{b} = \frac{a!}{b!\,(a-b)!}}$$

$$\frac{(n+1)!}{k!\,((n+1)-k)!} \overset{?}{=} \frac{n!}{(k-1)!\,(n-(k-1))!} + \frac{n!}{k!\,(n-k)!}$$

Fact: $\square! = \square(\square-1)!$

$$\frac{(n+1)!}{k!\,(n-k+1)!} \overset{?}{=} \frac{k\cdot n!}{k\boxed{(k-1)!}\boxed{(n-k+1)!}} + \frac{n!\,(n-k+1)}{\boxed{k!}\boxed{(n-k)!}\,(n-k+1)}$$

$$\frac{(n+1)!}{k!\,(n-k+1)!} \overset{?}{=} \frac{k\cdot n! + n!\,(n-k+1)}{k!\,(n-k+1)!}$$

$$\frac{(n+1)!}{k!\,(n-k+1)!} \overset{?}{=} \frac{\cancel{k\cdot n!} + n!\cdot n - \cancel{k\cdot n!} + n!}{k!\,(n-k+1)!}$$

$$\frac{(n+1)!}{k!\,(n-k+1)!} \overset{?}{=} \frac{n!\cdot n + n!}{k!\,(n-k+1)!} = \frac{n!\,(n+1)}{k!\,(n-k+1)!}$$

$$\frac{(n+1)!}{k!\,(n-k+1)!} \overset{?}{=} \frac{(n+1)!}{k!\,(n-k+1)!}$$

Yes

6.1 #5 $\quad a_n$ = # of ways to pay \$$n$

have \$1 coin, \$2 coin, \$5 coin, \$5 bill, \$10 coin, \$10 bill
\$20 bill, \$50 bill, \$100 bill

experiment

\$6 → How? $\quad$ \$1 coin • \$5 to go $\quad$ or (\$5 bill or coin) • \$1
or \$2 coin • \$4 to go

\$1 coin \$2 coin \$5 bill or coin

$$a_6 = (1)\cdot a_5 + (1)\cdot a_4 + (2)\cdot a_1$$

Induction: rec. relation

\$1 \$2 \$5 \$10 \$20 \$50 \$100

$$a_n = (1)a_{n-1} + (1)a_{n-2} + (2)a_{n-5} + (2)a_{n-10} + (1)a_{n-20} + (1)a_{n-50} + (1)a_{n-100}$$

$$\boxed{a_n = a_{n-1} + a_{n-2} + 2a_{n-5} + 2a_{n-10} + a_{n-20} + a_{n-50} + a_{n-100}}$$

Basis?:

$a_0 = 1$

$a_1 = a_0 + \cancel{a_{-1}}$

$a_1 = 1$

\$2 $a_2 = a_1 + a_0 + \cancel{2a_{-3}}$

$= 1 + 1 = 2$

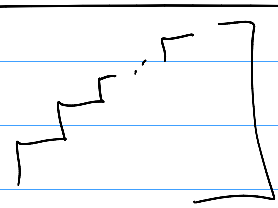
\$3 $a_3 = a_2 + a_1 = 2 + 1 = 3$

\$4 $a_4 = a_3 + a_2 = 3 + 2 = 5$

\$5 $a_5 = a_4 + a_3 + 2a_0 = 5 + 3 + 2(1) = 10$

#5 $a_{17} = ?$

8.1 #11



n-stairs take 1 step or 2 steps per leg motion

$$a_n = (1)a_{n-1} + (1)a_{n-2}$$

Basis: $a_0 = 1$ $1, 1, 2, 3, 5, 8, \ldots$

$a_1 = 1$

Modify: 1 step left or right leg

2 steps only with left leg

$$a_n = (2)\,a_{n-1} + (1)\,a_{n-2} \qquad a_0 = 1$$

left | right $\qquad$ 2 step $\qquad a_1 = 2$

$1, 2, 5, 12, 29, \ldots$

---

last 3

$$a_n = 2a_{n-1} + a_{n-2}$$

or

$$a_n = a_{n-1} + a_{n-2}$$

or

$$a_n = a_{n-1} + a_{n-2} + 2a_{n-5} + 2a_{n-10}$$

---

go from open rec. relation to a closed formula

ex $\boxed{a_0 = 1 \quad a_n = 2a_{n-1}} \to a_n = 2^n \quad n = 0, 1, 2, \ldots$

$1, 2, 4, 8, \ldots, a_{2000}$?

×2

?

$$\boxed{a_{2000} = 2^{2000}}$$

tech #1 guess - check

tech #2 iteration

8.2 advanced guess-check

if recurrence relation looks like (pattern)

then answer is ( ).

Pattern $$\boxed{a_n = c_1 a_{n-1} + c_2 a_{n-2} + c_3 a_{n-3} + \dots + c_k a_{n-k}}$$

linear homogeneous recurrence relation of degree k
with constant coeff.

guess: $a_n = r^n$ for some constant r.

check to find r

$$r^n = c_1 r^{n-1} + c_2 r^{n-2} + \dots + c_k r^{n-k}$$
$$1 = c_1 r^{-1} + c_2 r^{-2} + \dots + c_k r^{-k}$$
$$1 = \frac{c_1}{r} + \frac{c_2}{r^2} + \dots + \frac{c_k}{r^k}$$
$$r^k = c_1 r^{k-1} + c_2 r^{k-2} + \dots + c_k$$

$$\rightarrow \boxed{r^k - c_1 r^{k-1} - c_2 r^{k-2} - \dots - c_k = 0}$$

has k roots $r_1 = ?$
$r_2 = ?$
$\vdots$
$r_k = ?$

(ex) $a_n = a_{n-1} + 6a_{n-2}$ **Soln:** $a_n = r^n$?

$\rightarrow$ $r^2 - 1r - 6 = 0$ 2 solns
$(r-3)(r+2) = 0$
$r_1 = 3$ $r_2 = -2$ $a_n = (3)^n$ $a_n = (-2)^n$

$$\boxed{a_n = c_1(3)^n + c_2(-2)^n}$$

Note: putting multiple solutions together

$a_n = \boxed{\text{linear homogeneous rec. rel. of deg } k \text{ w/ const coeff}}$

char. poly $= 0$

solved

roots: $r_1 = \;, r_2 = \;, r_3 = \; \ldots, r_m =$

multiplicity

ex

$a_n = \boxed{}$

poly $= 0$

$\rightarrow (r-2)(r-2)(r+1)(r+1)(r+1)(r-3) = 0$

roots:	$r = 2$	$r = -1$	$r = 3$
multiplicity	2	3	1

$$\boxed{\begin{aligned} a_n = &\ (c_1 + c_2 n)(2)^n \\ &+ (c_3 + c_4 n + c_5 n^2)(-1)^n \\ &+ (\quad c_6 \quad)(3)^n \end{aligned}}$$

---

ex $f_n = f_{n-1} + f_{n-2}$ $\quad f_0 = 0 \quad f_1 = 1$

$0, 1, 1, 2, 3, 5, 8, \ldots \quad \underline{f_{1000}}$

$r^2 - 1r - 1 = 0$

$r = \dfrac{1 \pm \sqrt{1+4}}{2}$ $\quad r_1 = \dfrac{1+\sqrt{5}}{2}$ $\quad r_2 = \dfrac{1-\sqrt{5}}{2}$

$f_n = c_1 \left(\frac{1+\sqrt{5}}{2}\right)^n + c_2 \left(\frac{1-\sqrt{5}}{2}\right)^n$

$$f_0 = 1 \rightarrow \quad 1 = C_1 + C_2 \rightarrow C_2 = 1 - C_1$$

$$f_1 = 1 \rightarrow \quad 1 = C_1\left(\frac{1+\sqrt{5}}{2}\right) + C_2\left(\frac{1-\sqrt{5}}{2}\right)$$

$$2 = C_1(1+\sqrt{5}) + (1-C_1)(1-\sqrt{5})$$

$$2 = C_1(1+\sqrt{5}) + (1-\sqrt{5}) - C_1(1-\sqrt{5})$$

$$1+\sqrt{5} = C_1(1+\sqrt{5}) - C_1(1-\sqrt{5})$$

$$1+\sqrt{5} = C_1 \, 2\sqrt{5}$$

$$C_1 = \frac{1+\sqrt{5}}{2\sqrt{5}}$$

Exam 4 12 probs @ 10 pts 110 pts = 100%

See Sample exam 4

Note: Final = Comprehensive (Exam 1 to 4)

If Final % is greater than lowest exam, Lowest exam is replaced by Final %

Ex

70%	80%	62%	69%	71%
E1	E2	E3	E4	Final

result) 70%, 80%, 71%, 69%, 71%