Math 321

Q5 8.2 #11 $\boxed{L_0 = 2 \quad L_1 = 1}$ basis $\boxed{L_n = L_{n-1} + L_{n-2}}$ recurrence relation

$\{L_n\}$ 2, 1, 3, 4, 7, 11, 18, ...

$\{f_n\}$ 0, 1, 1, 2, 3, 5, 8, 13, ...

(1.2) Fibonacci Numbers $f_0 = 0 \quad f_1 = 1 \qquad f_n = f_{n-1} + f_{n-2}$

$\{f_n\}$ 0, 1, 1, 2, 3, 5, 8, 13, ...

#11 show $L_n = f_{n-1} + f_{n+1} \qquad n = 2, 3, 4, \ldots$

(a) pf: by induction:

Basis: prove 1^{st} case $(n=2)$

show $L_2 \stackrel{?}{=} f_1 + f_3$

$3 \stackrel{?}{=} 1 + 2$ true

Induction

$n=2$
$n=3$

assume k^{th} $\boxed{L_k = f_{k-1} + f_{k+1}}$

show $(k+1)^{st}$ $L_{k+1} \stackrel{?}{=} f_k + f_{k+2}$

$L_{k+1} = L_k + L_{k-1} = (f_{k-1} + f_{k+1}) + (f_{k-2} + f_k)$

I.H. $= f_k + f_{k+2}$ true

(b) $L_n = L_{n-1} + L_{n-2}$

$r^2 - r - 1 = 0 \qquad r = \dfrac{1 \pm \sqrt{1+4}}{2}$

$r_1 = \dfrac{1+\sqrt{5}}{2} \qquad r_2 = \dfrac{1-\sqrt{5}}{2}$

$$L_n = (a)\left(\frac{1+\sqrt{5}}{2}\right)^n + (b)\left(\frac{1-\sqrt{5}}{2}\right)^n$$

$$L_n = a\left(\frac{1+\sqrt{5}}{2}\right)^n + b\left(\frac{1-\sqrt{5}}{2}\right)^n$$

$L_0 = 2$

$L_1 = 1$

$$\begin{cases} 2 = a + b \leftarrow b = 2 - a & b = 1 \\ 1 = a\left(\frac{1+\sqrt{5}}{2}\right) + b\left(\frac{1-\sqrt{5}}{2}\right) \end{cases}$$

$$1 = a\left(\frac{1+\sqrt{5}}{2}\right) + (2-a)\left(\frac{1-\sqrt{5}}{2}\right)$$

$$2 = a(1+\sqrt{5}) + (2-a)(1-\sqrt{5})$$

$$\cancel{2} = \cancel{a} + \sqrt{5}a + \cancel{2} - 2\sqrt{5} - \cancel{a} + \sqrt{5}a$$

$$0 = 2\sqrt{5}a - 2\sqrt{5}$$

$$a = 1$$

$$\boxed{L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n}$$

$L_0 = 2 \quad L_1 = 1$

$L_n = L_{n-1} + L_{n-2}$

$\{L_n\}$ 2, 1, 3, 4, 7, 11, ...

$n=0$ $n=1$ $n=2$

Exam 4

$\boxed{\text{12 probs @ 10pts / 110pts = 100\%}}$

Basics of Counting

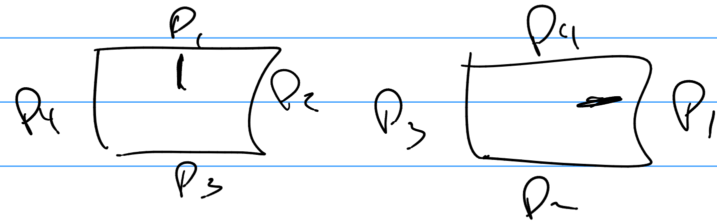
- Sum Rule "or" in the task to count
- Product Rule "and" in the task to count

(overcount)

- Sum Rule has intersection → Inclusion / Exclusion

ex $|A \cup B \cup C| = |A| + |B| + |C|$
$- |A \cap B| - |A \cap C| - |B \cap C|$
$+ |A \cap B \cap C|$

- Division Rule (over count of product rule)



Problems 1-3 Basic Rules

(1) $\overset{\text{and}}{D_1}\overset{\text{and}}{D_2}\overset{\text{and}}{L_1}\overset{\text{and}}{L_2}\overset{\text{and}}{L_3}L_4$ or $D_1D_2D_3D_4L_1L_2$ or $D_1D_2D_3D_4D_5D_6$

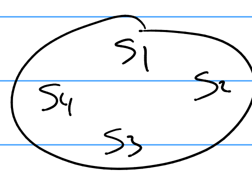
$10\cdot 10\cdot 26\cdot 26\cdot 26\cdot 26$

$$\left|10^2\, 26^4 + 10^4\cdot 26^2 + 10^6\right|$$

(2) overcount (Inclusion / Exclusion) 2 sets

$$|A\cup B| = |A| + |B| - |A\cap B|$$

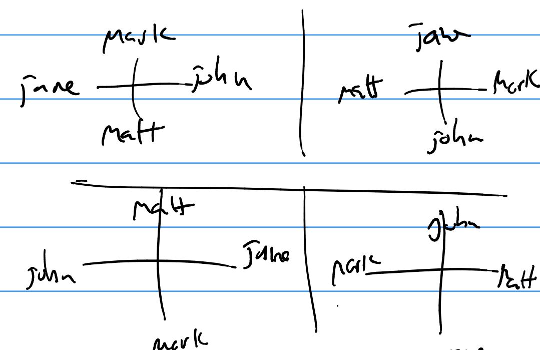
(3) Division Rule



10 people
seat 4 people ← task

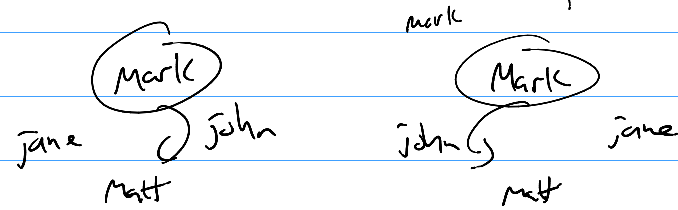
Step 1 pick 4 $P(10,4) = \frac{10!}{6!} = 10\cdot 9\cdot 8\cdot 7$

Step 2 overcount? a) symmetry.



divide by 4 →

b) rotation:



divide by 2

$$|\text{task}| = \boxed{\frac{10\cdot 9\cdot 8\cdot 7}{4\cdot 2}} = \boxed{\frac{\frac{10!}{6!}}{4\cdot 2}}$$

Pigeonhole Principle: (principle and generalized principle)

1) have $K+1$ or more objects and K boxes then at least one box has at least 2 objects

(generalized) have N objects and K boxes then at least one box has at least $\lceil \frac{N}{K} \rceil$ objects

1 problem

Ans: $200 \cdot 300 \cdot 10 = 600000$

Prob: $\left\lceil \frac{N}{600{,}000} \right\rceil = 4$ $\left\lceil \frac{N}{3} \right\rceil = 4$

$N = 1 + 3 \cdot 600{,}000$

$\boxed{N = 1{,}800{,}001}$

$C(n,r) = \binom{n}{r} = \frac{n!}{r!\,(n-r)!}$ $P(n,r) = \frac{n!}{(n-r)!}$

2 probs

ex 5 guys and 7 girls

a) choose 3 people $\binom{12}{3} = \frac{12!}{3!\,9!}$

b) pick 3 people $P(12,3) = \frac{12!}{9!}$

→ c) choose 5 but more girls than guys

5 girls 0 guys or 4 girls 1 guy or 3 girls 2 guys

$\binom{7}{5}\binom{5}{0} + \binom{7}{4}\binom{5}{1} + \binom{7}{3}\binom{5}{2} = \boxed{}$

use ! notation

Binomial th^n

$$(\square + \triangle)^n = \frac{n!}{n!0!}\square^n\triangle^0 + \frac{n!}{(n-1)!1!}\square^{n-1}\triangle^1 + \frac{n!}{(n-2)!2!}\square^{n-2}\triangle^2 + \dots + \square^0\triangle^n$$

1 prob

use

(ex) $\left(2x - \frac{1}{x}\right)^4 =$ 5 terms

(ex) $\left(2x - \frac{3}{x^2}\right)^{107} \longrightarrow$ tell me the 42^{nd} term

$$\frac{107!}{66!\,41!}(2x)^{66}\left(-\frac{3}{x^2}\right)^{41} =$$

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prove $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$

Combinatorial Proof?

Algebra?

$\triangleright$ n red blocks and 1 special gold block

task: choose k blocks

tech #1 $\binom{n+1}{k}$ choose k from the $n+1$ blocks

tech #2 how many have gold block or do not have it

$$\underbrace{1 \cdot \binom{n}{k-1}}_{\text{have gold}} + \binom{n}{k}$$

Count same task so $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$

recurrence relations (4 probs)

1. find basis (+) relation
2. find function by iteration
3. } 8, 2
4. }

$$\boxed{a_n = a_{n-1} + 3a_{n-2} - 2a_{n-3}}$$

$$r^3 - r^2 - 3r + 2 = 0$$

$$(r^2 - 2r + 1)(r + 2)$$

$$(r-1)^2 (r+2) = 0$$

mult. 2 → $r = 1$ $r = -2$

$$\boxed{a_n = (a + bn)(1)^n + (c)(-2)^n}$$