Math 321

Q's

Final Exam

- Exam 1 → 4 probs
- Exam 2 → 4 probs
- Exam 3 → 4 probs
- Exam 4 → 4 probs

@ 10 pts each
150 pts = 100%

Note:

(#) "on" the test ← variation? / of type? / exact?

~~(#)~~ not on test

?(#) study it (maybe on test)

How to Study:

- Tonight → Exam's 1, 2
- Sat → Exam's 2, 3
- Sun → Exam 4
- Monday → light study / rest

Math 321 ... Final Review

Exam 1

1) Construct the truth table everyone should know.

2) Express "For the mouse to defeat the cat it is sufficient that the mouse drinks lots of coffee or the cat is sleepy" using propositional symbols and logical operators.

* If here it may include #6

3) Construct the truth table for $p \wedge (q \to p)$.

4) Show that the statements $(p \to q) \wedge (p \to r)$ and $p \to (q \wedge r)$ are logically equivalent.

by discussion? by truth table? by other log. equiv?

5) Use logical equivalences to show that $(p \wedge q) \to p$ is a tautology.

6) a) Let $S(u)$ mean that "u is silly," $F(v)$ mean that "v is fast," and $B(a, b)$ mean that "a has beat b in a race", where the universe of discourse for every variable consists of all children. Express $\exists x(F(x) \wedge \forall y(S(y) \to B(x, y)))$ by a simple English sentence.

b) Use quantifiers and propositional functions to express "Every Math 321 student kid has eaten cheese or some type of meat".

7) The following argument is not valid. "You do not do every problem in the book or you learn discrete mathematics. You learned discrete mathematics. Therefore, you did every problem in the book." Explain why it isn't valid..

"same type" & problem

8) Come up with a valid conclusion for the set of premises: "If I eat at bedtime, then I can not sleep." "I can not sleep if there is music playing." "I slept last night." "Not sleeping is sufficient for me to not pass Math 321." Explain your answers.

9) Prove if a is an odd number then $a^2 + 3$ is a multiple of 4.

10) Prove that $\sqrt{2}$ is irrational. (Include the proof of the needed lemma)

11) For the integers 2,3,4,... Prove: if $n^2 < 2^n$, then $n > 4$. (contraposition, cases)

12) Show that there exist irrational numbers x and y such that, x^y is rational. (non-constructive existence proof)

EXAM 2

1) Use set builder notation and roster forms to represent each of the following sets. The set A is even integers from 2 to 9, the set B is integers that are a multiple of 3 from -5 to 7, and the set C is integers that are prime. The universe of discourse is all integers from -6 to 10. And then illustrate all the sets and the universe of discourse with a single Venn Diagram.

2) For $A = \{s, c\}$ and $B = \{3\}$ find ...

a) $B \times A$.

b) and then $P(B \times A)$.

3) Show $(A - B) \cup A = A$ using any method.

membership tables
set builder notation (+) logic
discussion $A = B$ is $A \subseteq B$ & $B \subseteq A$
Venn diagram

4) Show $(A \cap B \cap C) \subseteq (B \cap C)$.

5) Show that even if f and $f \circ g$ are onto, g does not have to be onto.

6) a) Find a function $f : \mathbb{Z}^+ \to \mathbb{Z}$ where f is not one-to-one and is onto.

b) Find a function $f : \mathbb{Z}^+ \to \mathbb{Z}$ where f is one-to-one and is onto.

7) Sequences ...

a) List the first 5 terms of the sequence $a_0 = 1, a_1 = 0$ and $a_n = 3a_{n-1} + 1a_{n-2}$.

b) Find some formulae for the sequence: -2, 4, 10, 16, 22, ...

8) Use telescoping sums to verify the formula

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

9) Find the value of the given sum using the formula from problem 8 ...

$$\sum_{k=3}^{111} 4k + 2$$

10) Prove that $\mathbb{Q}$ is countable.

11) Prove that $\mathbb{R}$ is uncountable.

12) Suppose that Hilbert's Grand Hotel is fully occupied, but you need to empty all the odd numbered rooms because they smell oddly and need maintenance. As the manager, what instructions would you give to the guests to move them around so they all still have a room?

Exam 3

1) Given $a, b,$ and c are integers, Show that if $a|b$ and $a|c$, then $a|2b - 3c$.

2) a) Find $-22 \operatorname{div} 6$ and $-22 \bmod 6$

b) Find $22 \operatorname{div} 6$ and $22 \bmod 6$

3) List two negative integers and two positive integers that are congruent to -3 modulo 5.

4) Evaluate $(2^{502} + 71)^{100} \bmod 3$.

5) Perform the requested operations ...

a) $(1, 2, 3)_7 + (4, 5)_7$ using only base 7 numbers. Write your answer in both base 7 and base 10.

b) $(1, 4)_7 \times (2, 5)_7$ using only base 7 numbers. Write your answer in base 7.

6) Find the prime factors, the gcd, and the lcm of 140 and 75 using prime factorization. Don't multiply out the product of primes for your answers.

7) Find the gcd of 140 and 75 using Euclid's Algorithm.

8) Given the affine-shift function: $f(p) = (7p + 3) \bmod 13$ find the decryption function $f^{-1}(c)$.

9) Given the key of $e = 5$ and $n = 221$ of a public key encryption. Find the decryption function $f^{-1}(c)$.

10) Prove for $n = 1, 2, 3, \ldots$ that

$$\frac{1}{2} + \frac{1}{4} + \ldots + \frac{1}{2^n} = 1 - \frac{1}{2^n}$$

11) Prove for $n = 2, 3, 4, \ldots$ that

$$1 + \frac{1}{4} + \frac{1}{9} + \ldots + \frac{1}{n^2} < 2 - \frac{1}{n}$$

12) Prove all integers $n \geq 2$ are prime or can be written as a product of primes.

Exam 4

1) How many passwords can be made where it uses either 10 digits, or 7 upper case letters, or starts with a upper case letter followed by five letters or digits?

2) How many bit strings of length eight either start with a 1 bit or end with the two bits 00?

3) How many ways are there to seat five people from a group of nine around a circular table? How many ways if it doesn't matter if the order is clockwise or counter-clockwise?

4) A company stores products in a large warehouse. Storage bins in the warehouse are specified by their aisle, location in the aisle, and shelf. There are 100 aisles, 200 horizontal locations in each aisle, and 12 shelves in each horizontal location throughout the warehouse. What is the least number of products the company can have so that at least three products must be stored in the same bin?

5) Nine people (5 Math majors and 4 CS majors) show up for a basketball game.

a) How many ways are there to choose 5 players to play?

b) How many ways are there to pick 5 players to play?

6) How many committees of seven people chosen from 16 people (9 Math faculty and 7 CS faculty) have more more Math faculty committee members than CS faculty members?

7) What is the expansion of $(x^2 + \frac{1}{x^3})^5$? Leave each coefficient in factorial notation, but write the variable x to a specific power for each term.

? 8) Prove $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$

$a_0 = 1$

$a_n = 2a_{n-1} + 2a_{n-2} + a_{n-3}$

9) Find a recurrence relation with initial conditions for the number of ways to walk up stairs with n-steps if you can take one step using either or your right leg or left leg. Or you can go up two steps using either your right leg or left leg. Or you could go up three steps with one large jump. After you have the basis values and recurrence relation write the first 5 values of the sequence.

1, 2, 5, 15, 42,

Use $\sum_{k=0}^{n} ar^k = a\left[\frac{r^{n+1}-1}{r-1}\right]$

10) Use iteration to solve the recurrence relation $a_n = 3a_{n-1} + 1$ where $a_0 = 1$.

11) Solve $a_n = -2a_{n-1} + 8a_{n-2}$.

? 12) Solve $a_n = -2a_{n-1} + 4a_{n-2} + 8a_{n-3}$.

$r^3 + 2r^2 - 4r - 8 = 0$

$r^2(r+2) - 4(r+2) = 0$

$(r^2-4)(r+2) = 0$

$r = -2 \quad r = -2 \quad r = 2$ ⇒ etc