# Math 322

## Discrete 2

toys + rules = Math

Sets + operations → review!

prop. function ← "Logic"

$S = \{ e \mid P(e) \}$

**Note:** $A_1 \times A_2 \times \cdots \times A_n = \{ \underbrace{(a_1, a_2, \ldots, a_n)}_{n\text{-tuple}} \mid a_1 \in A_1 \wedge a_2 \in A_2 \wedge \cdots \wedge a_n \in A_n \}$

cross product

Set of $\boxed{\text{All}}$ ordered n-tuples from $A_1, A_2, \ldots, A_n$

ex

$A = \{1, 2\}$

$B = \{0, \square, \Delta\}$

$C = \{ \smile, \frown \}$

$A \times B \times C = \{ (1, 0, \smile), (1, 0, \frown), (1, \square, \smile), (1, \square, \frown), \ldots, (2, \Delta, \frown) \}$

$$|A \times B \times C| = |A||B||C| = 2 \cdot 3 \cdot 2 = 12$$

→ What about subsets of $A \times B \times C$?

$P(A \times B \times C)$ = set of all subsets of $A \times B \times C$

$= \{ \emptyset$ — take no tuples

$\{(1, 0, \smile)\}, \ldots, \{(2, \Delta, \frown)\},$ — take 1 tuple

$\vdots$

$\{ (1, 0, \smile), \ldots, (2, \Delta, \frown) \} \}$ ← take all

$$|P(A \times B \times C)| = 2^{|A \times B \times C|} = 2^{12}$$

So <u>some</u> subset of $A \times B \times C$ acts as a relationship between $A, B, C$.

(ex) $R = \{ (1, 0, \smile), (1, \square, \smile), (1, \Delta, \smile), (2, \Delta, \smile) \}$

**Def:** any subset of $A_1 \times A_2 \times \cdots \times A_n$ is a relationship between the sets $A_1, A_2, \ldots, A_n$.

→ call it an <u>n-ary</u> relation.

---

**Applications:** ① restrict sets to two.

<u>Study</u> sets of ordered pairs.

a binary relation is subset $A \times B = \{ (a,b) \mid a \in A \wedge b \in B \}$

**Def:** A binary relation <u>from</u> A <u>to</u> B is a subset of $A \times B$

(ex) So $R$ is binary relation from $A$ to $B$

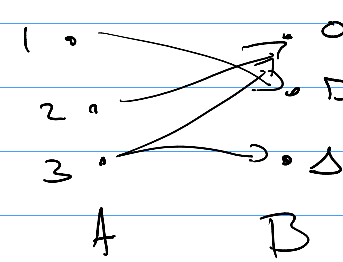
<u>and</u> $(a,b) \in R$ : $a$ is related to $b$ : $aRb$

$(a,b) \notin R$ : $a$ is not related to $b$ : $a \not{R} b$

**Visualize** ① <u>roster</u> ex $R = \{ (1, \square), (2, 0), (3, \Delta), (3, 0) \}$

② <u>arrow diagram</u> $A = \{1, 2, 3\}$

$B = \{0, \square, \Delta\}$

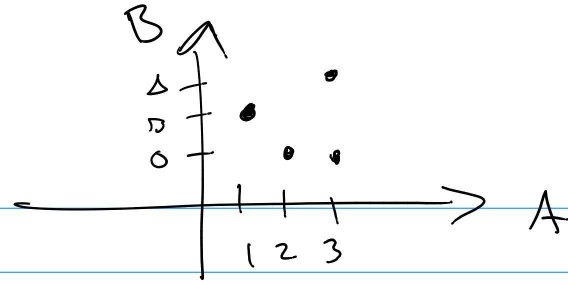



③ <u>tally table</u>

| | | B | | |
|---|---|---|---|---|
| | $R$ | $0$ | $\square$ | $\Delta$ |
| | 1 | | X | |
| A | 2 | X | | |
| | 3 | X | | X |

$\rightarrow \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

(a) graph



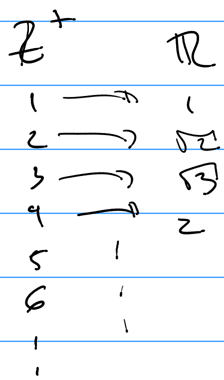

Note: if we restrict $R$ from $A$ to $B$ such that every $a \in A$ goes to exactly one $b \in B$.

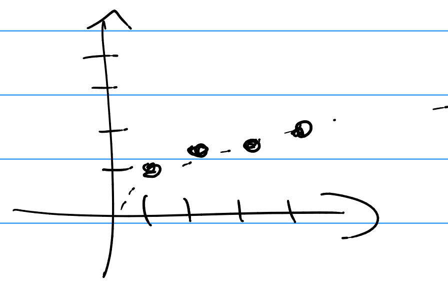
---

So, we have a new "toy" to play with.. $\boxed{\text{binary relations}}$

(ex) $R$ is from $\mathbb{Z}^+$ to $\mathbb{R}$

$$R = \{ (a,b) \mid b = (a)^{1/2} \}$$



---

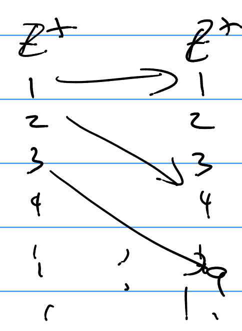
B Try to categorize the toys or relations

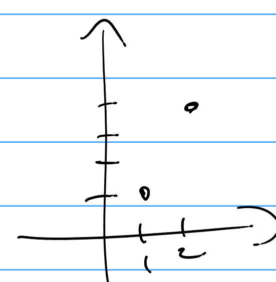
(ex) ① Functions

if we study $R$ on $A$ to $A$ (subset of $A \times A$)

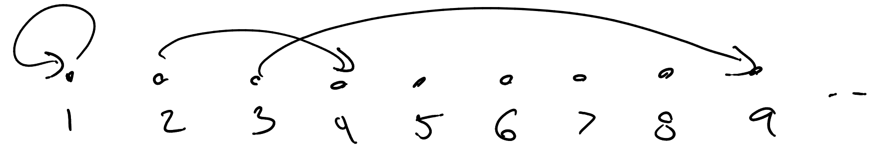
Call these $\boxed{R \text{ on set } A}$

(ex) $R$ on $\mathbb{Z}^+$ to $\mathbb{Z}^+$ (ex)



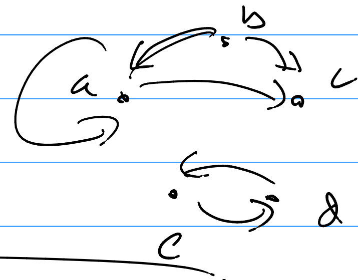



write $\mathbb{Z}^+$ once

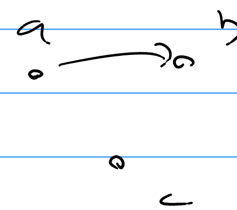



So $R$ on $A = \{a, b, c, d, e\}$

$R = \{(a,a), (a,c), (b,a), (b,c), (d,e), (e,d)\}$




## Properties of R on set A




① reflexive $\forall e (eRe)$

② irreflexive $\forall e (e \not R e)$

③ symmetric $\forall e_1 \forall e_2 (e_1 R e_2 \rightarrow e_2 R e_1)$

④ anti symmetric $\forall e_1 \forall e_2 (e_1 R e_2 \wedge e_2 R e_1 \rightarrow e_1 = e_2)$

$\equiv \forall e_1 \forall e_2 (e_1 \neq e_2 \rightarrow (e_1 \not R e_2 \vee e_2 \not R e_1))$

$\equiv \neg \exists e_1 \exists e_2 (e_1 R e_2 \wedge e_2 R e_1 \wedge e_1 \neq e_2)$