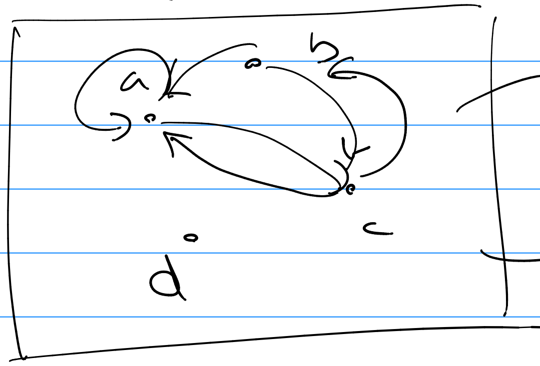
Math 322

Relations: $R: A \to A$ R is subset of $A \times A$

$R = \{(a,a), (a,c), (b,a), (b,c), (c,a), (c,b)\}$
on $A = \{a,b,c,d\}$



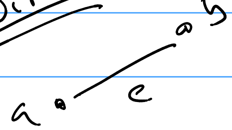
Vertices: V, need to be non-empty

Edges: E, "connect" vertices

New toy: Graph. $G = (V, E)$

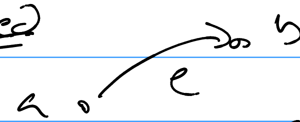
V is a non-empty set of vertices
E is a set of pairs of vertices called edges.

undirected



$e = \{a, b\}$

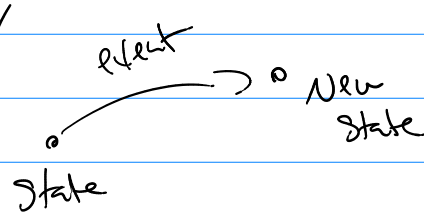
directed



$e = (a, b)$

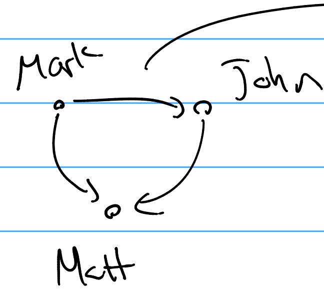
Ch 10 Graph theory

- applications



State machine

- brackets
- influence graphs

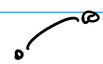


Mark "influences" John

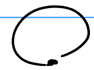
- "Social" Graphs : "friend" , "follow"

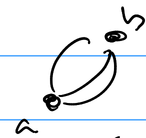
"friend" – undirected ; "follow" – directed

Types

undirected 

① Simple : no loops, no multiple edges

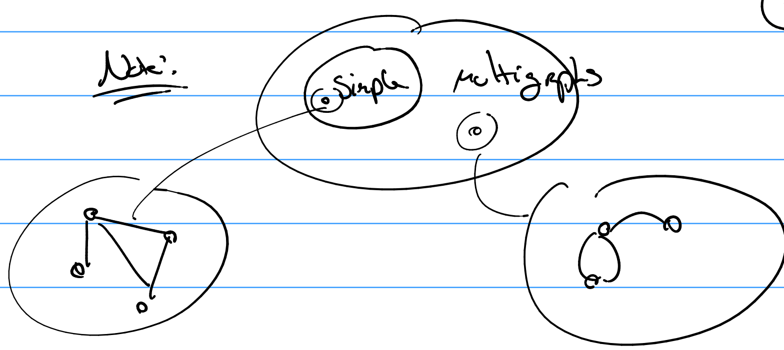
loop? 

multiple edge? 

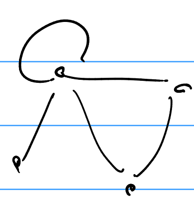
$e_1 = \{a,b\}$
$e_2 = \{a,b\}$
$e_3 = \{a,b\}$

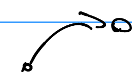
② multigraph : no loops

Note:



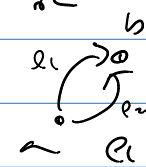
③ pseudograph

ex 

directed 

① Simple directed
- no loops
- no mult. edges

loop? 

mult. edge? 

$e_1 = (a,b)$
$e_2 = (a,b)$

② Directed Multigraph

has loops? ok

has mult. edges? ok

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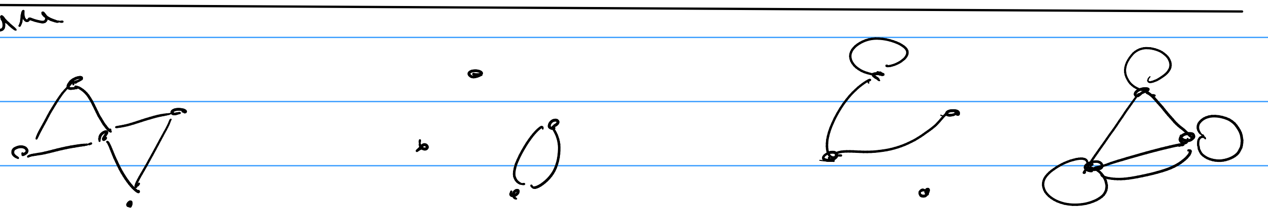
call by best name

ex



Simple | multigraph | psuedograph | pseudograph

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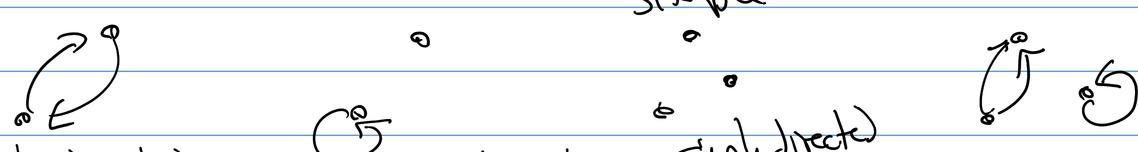
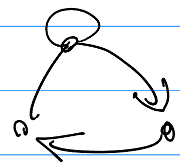
ex



Simple directed | Directed Multigraph | Simple / simple directed | Directed Multigraph have both directed and undirected edges : Mixed Graph



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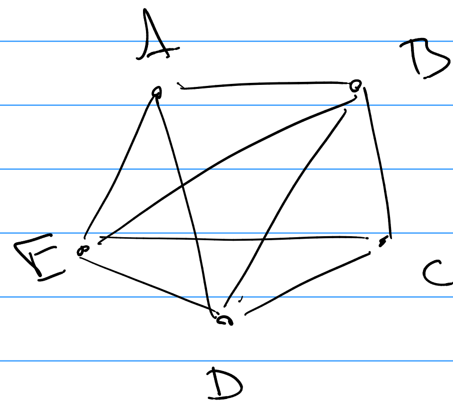
ex) Intersection Graph

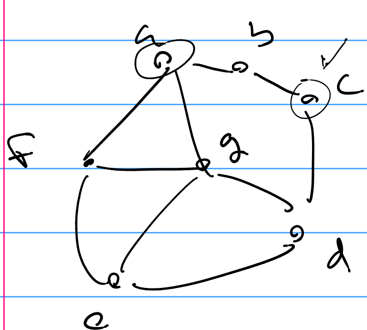
$A = \{a, b, c, d, e, f, g\}$
$B = \{a, e, i, o, u\}$
$C = \{m, n, o, p\}$
$D = \{p, e, a, r\}$
$E = \{m, a, r, k\}$



10.2 Terms and Special Graphs

Undirected $v_1 \overset{e}{\text{———}} v_2$

- e is incident to v_1, v_2
- v_1, v_2 are adjacent
- v_1, v_2 are connected by e



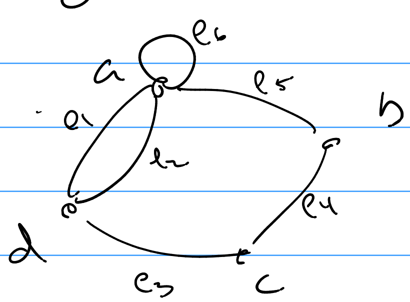
- neighbors

ex) neighbors of g : a, f, e, d

- neighborhood of v N(v)

ex) $N(g) = \{a, f, e, d\}$

+ degree of a vertex : deg(v) = number of incident edges except loops = 2



deg(a) = 5
deg(b) = 2
deg(c) = 2
deg(d) = 3

properties of $\deg(v)$

① $\deg(v) = 0$ call v isolated

② $\deg(v) = 1$ call v pendant

thm $\sum_{v \in V} \deg(v) = 2|E|$

thm G will always have an even number of odd degree vertices.

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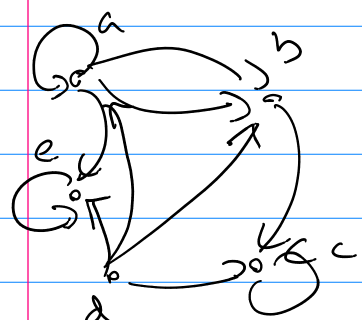
directed graphs

$v_1 \xrightarrow{e} v_2$ $e = (v_1, v_2)$

- v_1 is the initial vertex
- v_2 is the terminal vertex
- v_1 is adj. to v_2
- v_2 is adj. from v_1

- in-degree: $\deg^-(v)$ = # of times v is a terminal vertex
- out-degree: $\deg^+(v)$ = # of times v is an initial vertex

thm $\sum \deg^+(v) = \sum \deg^-(v) = |E|$



$\deg^-(a) = 2$ $\deg^+(a) = 4$

$\deg^-(b) = 3$ $\deg^+(b) = 1$

$\deg^-(c) = 3$ $\deg^+(c) = 1$

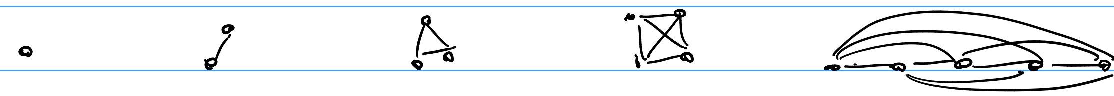
$\deg^-(d) = 0$ $\deg^+(d) = 4$

$\deg^-(e) = 3$ $\deg^+(e) = 1$

Graphs to Know

Special Undirected Simple Graphs

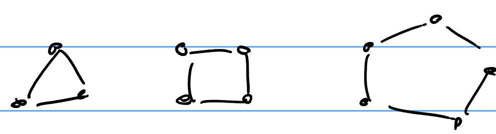
① Complete Graph : K_n , $n \geq 1$, n : # of vertices



K_1 K_2 K_3 K_4 K_5

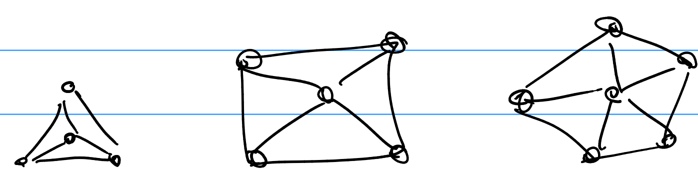
$E = \frac{1(0)}{2}$ $\frac{2(1)}{2}$ $\frac{3(2)}{2}$ $\frac{4(3)}{2}$ $\frac{5(4)}{2}$

② Cycle : C_n , n = # of vertices , $n \geq 3$



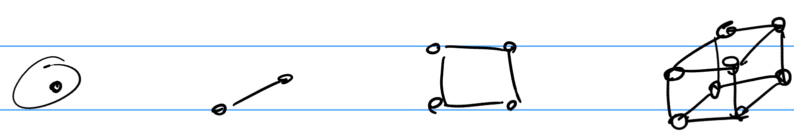
C_3 C_4 C_5

③ Wheel ; W_n, C_n (+) one extra vertex (axle) , $n+1$ = # of vertices
$n \geq 3$



W_3 W_4 W_5

n-cube Q_n (n^{th} dimensional cube)



Q_0 Q_1 Q_2 Q_3

Q_4