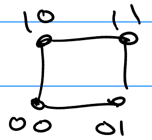
Math 322

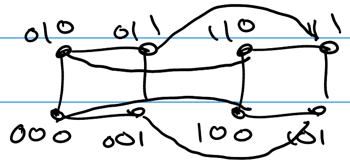
Know: K_n, C_n, W_n, Q_n (n^{th} dim. n-cube)


Q_0

Q_1



Q_2

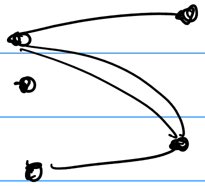


Q_3

- Bipartite Graphs: $V = V_1 \cup V_2$ ($V_1 \cap V_2 = \phi$)

and all edges go between V_1 to/from V_2
(no connections within V_1 to V_1 or V_2 to V_2)

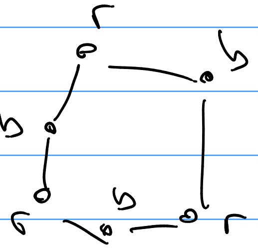
ex



Coloring thm

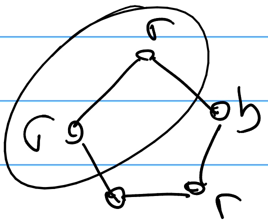
bipartite iff you can assign $v_i \in V$ one of two colors and no same color vertices are connected.

ex



C_6

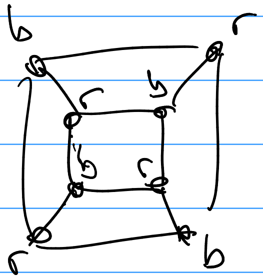
bipartite by coloring thm



C_5

not bipartite

ex Q_3



so Q_3 is bipartite.

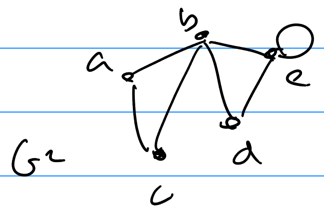
Still $G = (V, E)$ are two sets. (So we can do set ops)

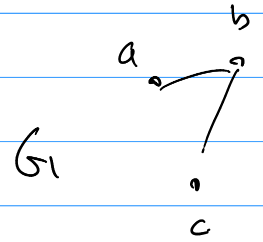
① $G_1 = (V_1, E_1)$ $G_2 = (V_2, E_2)$

$G_1 \cup G_2 = (V_1 \cup V_2, E_1 \cup E_2)$

② Subgraph $G_1 = (V_1, E_1)$ $G_2 = (V_2, E_2)$

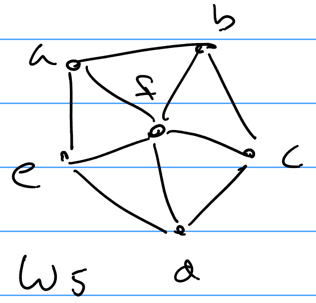
$G_1 \subseteq G_2$ $V_1 \subseteq V_2$ $\boxed{E_1 \subseteq E_2}$



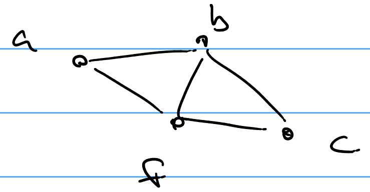


note: edges in E_1 must only use vertices of V_1

③ Subgraph induced by $V_1 \subseteq V_2$

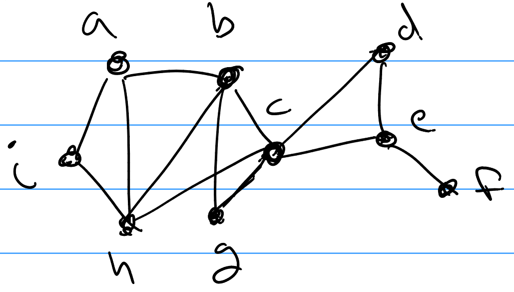


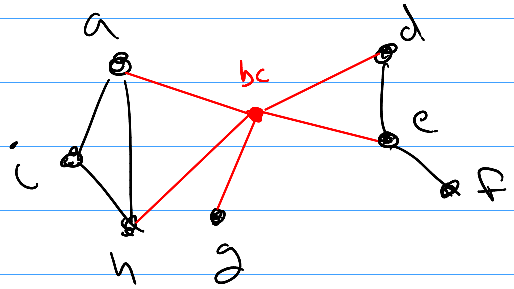
subgraph induced by $\{a, b, c, f\}$



(take all edges of the subset of vertices)

④ Edge Contraction



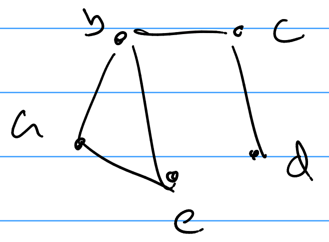


$\boxed{10.3}$ Representing graphs $G = (V, E)$

① Sets

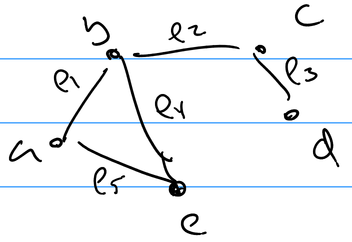
② undirected / directed graphs

③ lists (adj. lists)



	adj.
a	b, e
b	a, c, e
c	b, d
d	c
e	a, b

④ incident lists



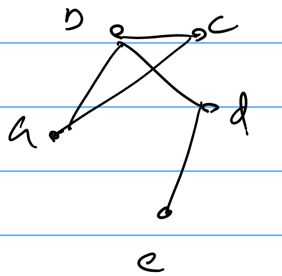
	incident with
a	e_1, e_5
b	e_1, e_2, e_4
c	e_2, e_3
d	e_3
e	e_4, e_5

⑤ Adjacency Matrix $A_G = [a_{ij}]$ $a_{ij} = \{ \#$ of edges between v_i v_j

simple graphs : all zeros - ones

otherwise : real valued matrix

ⓔⓧ



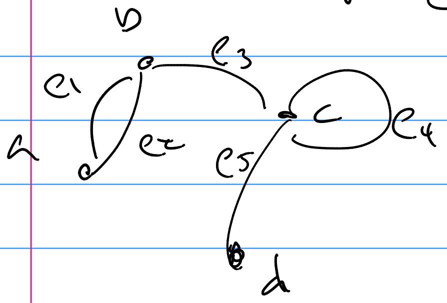
$$A_G = \begin{array}{c} a \\ b \\ c \\ d \\ e \end{array} \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

ⓔⓧ

$$A_G = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & 0 \end{bmatrix}$$

⑥ Incidence Matrix : $M_G = [M_{ij}]$.

$$M_{ij} = \begin{cases} 1 & \text{if } v_i \text{ is inc with } e_j \\ 0 & \text{otherwise} \end{cases}$$



$$M = V \begin{array}{c|ccccc} & & & E & & \\ & e_1 & e_2 & e_3 & e_4 & e_5 \\ a & 1 & 1 & 0 & 0 & 0 \\ b & 1 & 1 & 1 & 0 & 0 \\ c & 0 & 0 & 1 & 1 & 1 \\ d & 0 & 0 & 0 & 0 & 1 \end{array}$$

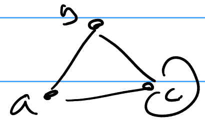
consider $\underset{(\text{people}\cdot\text{edges})(\text{edges}\cdot\text{people})}{M \cdot M^T} = \underset{\text{people, people}}{C}$

consider $M^T \cdot M = \underset{\text{edges, edges}}{D}$

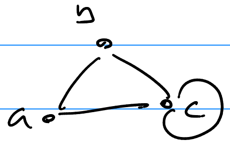
"Same"?   How to say G_1 is the same as G_2?

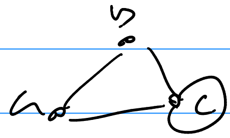
$G_1 = (V_1, E_2)$ $G_2 = (V_1, E_2)$

idea:

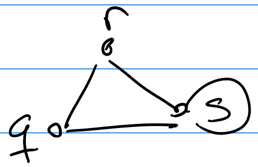


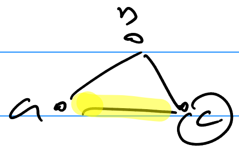
Same as



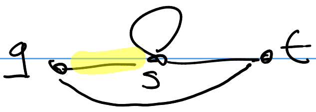


Same as





Same as



a ⟷ q
b ⟷ t
c ⟷ s

$\boxed{\text{Def}}$ G_1 and G_2 are isomorphic if there is a bijection from V_1 to V_2 that preserve edges.

$f: V_1 \to V_2$

there is an edge from a, b in V_1 iff there is an edge between $f(a), f(b)$

f is called the isomorphism.

How to show G_1, G_2 are isomorphic?

$$A_{G_1} = \begin{array}{c} v_1 \\ v_2 \\ \vdots \\ v_n \end{array}\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}, \quad \text{find } f \text{ the isomorphism}, \quad A_{G_2} = \begin{array}{c} f(v_1) \\ f(v_2) \\ \vdots \\ f(v_n) \end{array}\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$$

of V_1 ↑ ↑ of V_2

if $A_{G_1} = A_{G_2}$ then isomorphic

Use invariants to either say ① G_1, G_2 are not isomorphic

or ② help guess f.

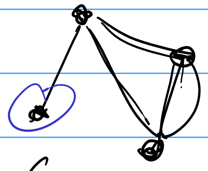
Invariants: (use properties of bijections)

① $|V_1| = |V_2|$

② $|E_1| = |E_2|$

③ degrees of neighborhoods are preserved.

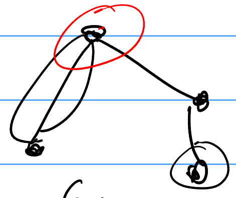
ex



G_1

$|V_1| = 4$

$|E_1| = 5$



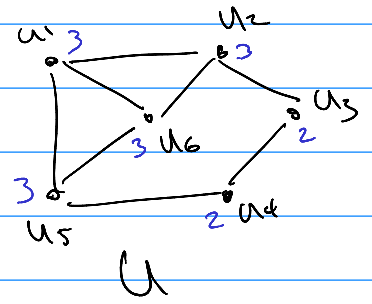
G_2

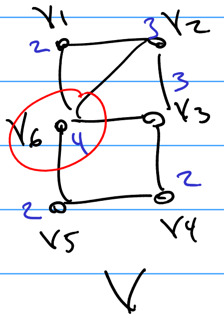
$|V_2| = 4$

$|E_2| = 5$

④ Paths are preserved

Ex





deg(v6) = 4

and no deg(u) is 4

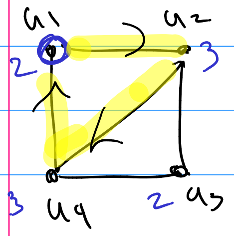
$\boxed{\text{not}}$ isomorphic

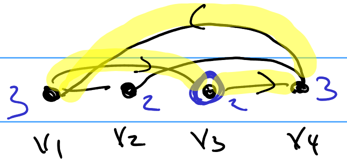
Vertices: 6

edges: 8

Vertices: 6

edges: 8





Vert.: 4

edges: 5

degrees look ok

guess f

f

$u_1 \to v_3$

$u_2 \to v_4$

$u_3 \to v_2$

$u_4 \to v_1$

check:

$$A_u = \begin{array}{c} u_1 \\ u_2 \\ u_3 \\ u_4 \end{array} \left[\begin{array}{c} \\ ? \\ \\ \\ \end{array} \right]$$

$$A_v = \begin{array}{c} v_3 \\ v_4 \\ v_2 \\ v_1 \end{array} \left[\begin{array}{c} \\ ? \\ \\ \\ \end{array} \right]$$