Math 322

10.5 $G = (V, E)$ can be represented by a graph or A_G

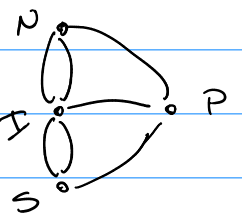
Euler Paths / Circuits

Hamilton Paths / Circuits

Königsberg Bridge Problem

Can I visit each bridge exactly once during a walk?

multigraph

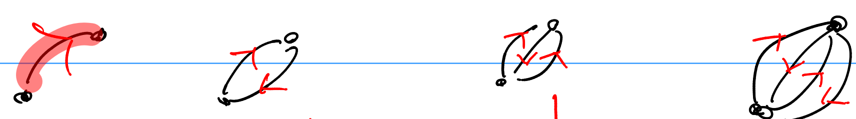


(Start = end) real question: does graph have a simple circuit that contains every edge (exactly once) (Euler Circuit)

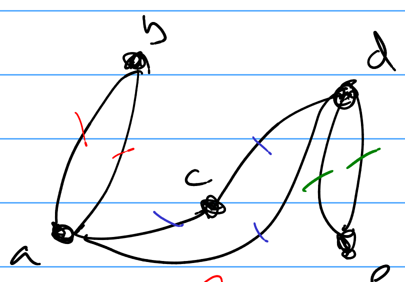
(Start ≠ end) real question: does graph have a simple path (Euler Path) (not circuit) that contains every edge.

Euler Circuit?

experiment:



no | Has Euler Circuit | no | Has Euler Circuit.



If all $\deg(v)$ are even I will have an Euler Circuit.

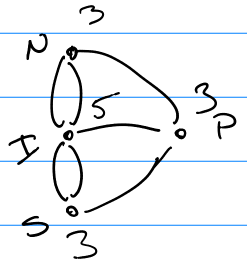
a, b, a

a, b, $\boxed{a, c, d}$, a

a, b, a, c, $\boxed{d, e, d}$, a

Thm: deg(v) is even for all vertices **iff** G has an Euler Circuit.

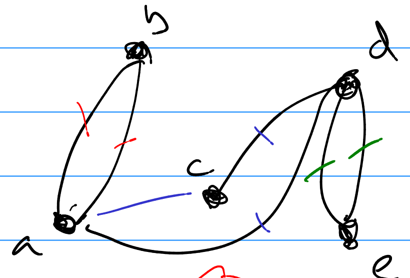
back to bridge problem

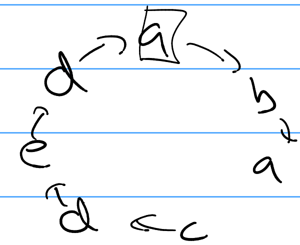


not all vertices are even so
No Euler Circuit.

Well.. what about Euler Path?

idea:



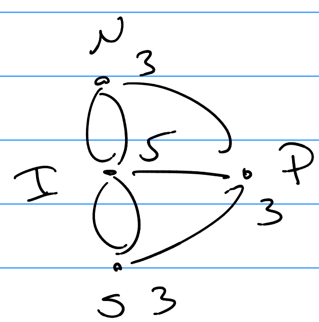


a, b, a
a, b, a, c, d, a
a, b, a, c, d, e, d, a

Thm: deg(v) has exactly two of odd degree (others are even) **iff** Euler path (not circuit) exists.

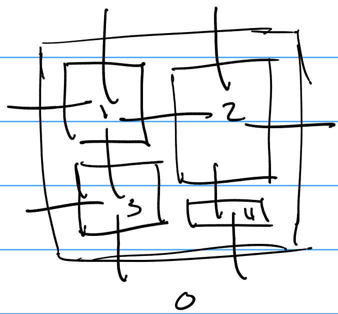
Note: the path starts/stops @ odd degree vertices.

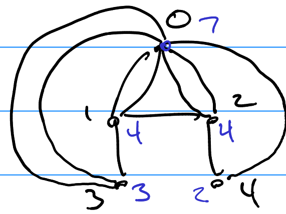
ex) back to bridge



No! there is no Euler path b/c we have more than two of odd degree.

but we can now do any city.





we have exactly two odd degree (7 and 3)

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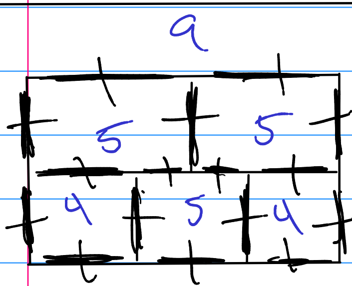
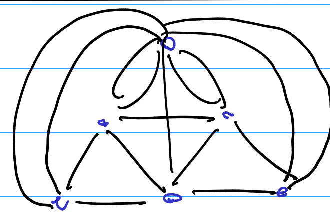
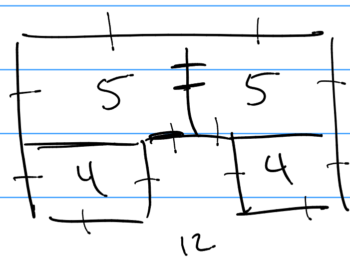
cut puzzle

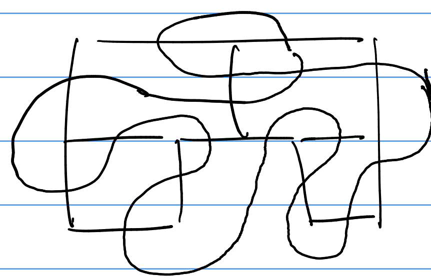




no soln

(b/c more than two odd degree)





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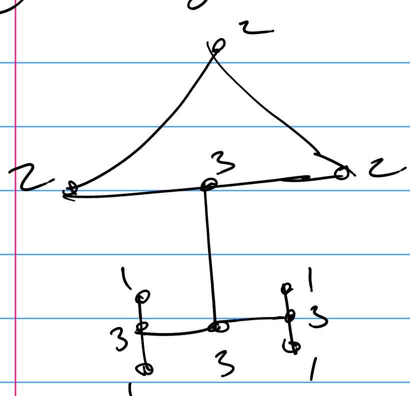
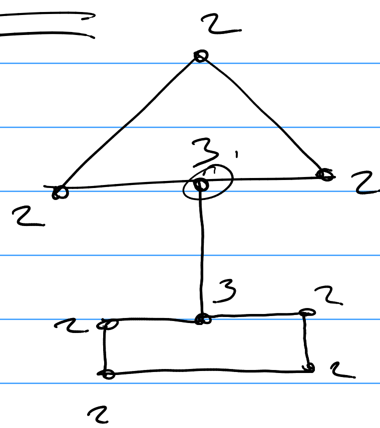
ex cutting tools (laser cutter)



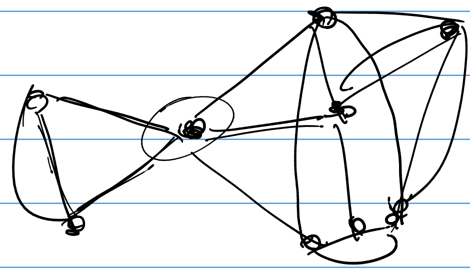


has euler path

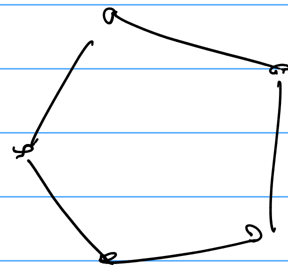
Hamilton Path: Simple path that has every vertex exactly once.

Hamilton Circuit: Simple circuit that has every vertex exactly once except first = last.

experiment

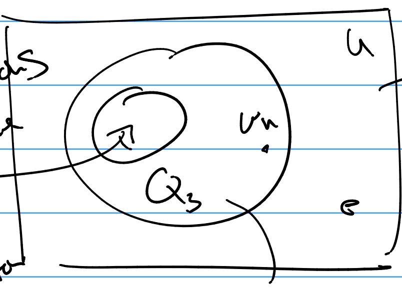


does not have Hamilton circuit.



Has Hamilton circuit.

Strong restrictions that allow me to say they have Hamilton circuit.



all graphs

have Hamilton Circuit

Thm **Dirac's** $G = (V, E)$ $|V| = n \geq 3$

if $\deg(v) \geq \frac{1}{2} \cdot n$ for all $v \in V$ $\rightarrow$ G has Hamilton Circuit.

Thm **Ore's** $G = (V, E)$ $|V| = n \geq 3$

for every v_1, v_2 non-adj. pairs of vertices

$\deg(v_1) + \deg(v_2) \geq n \rightarrow$ Hamilton Circuit.

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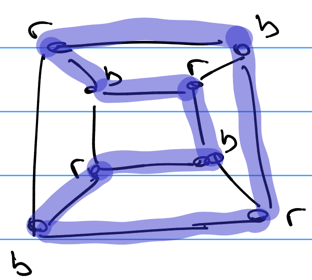
ex Q_3

$|V| = 8$



$\deg(v) = 3$

$\deg(r) = 3$

$\deg(b) = 3$

$\deg(v_1) + \deg(v_2) = 6$