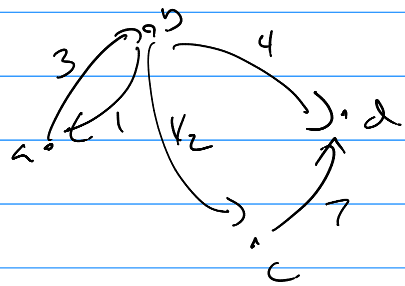
Math 322

10.6 **Weighted Graphs:** $G = (V, E)$ adds $w(e)$ weight function.



$w((a,b)) = 3$

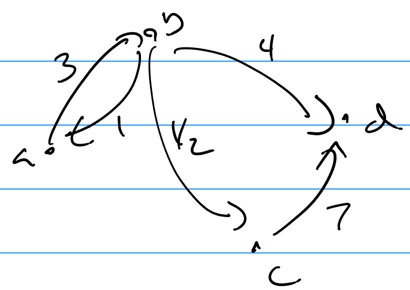
$w((b,a)) = 1$

no edge

$$\Rightarrow \quad A_G = \begin{bmatrix} 0 & 3 & 0 & 0 \\ 1 & 0 & 1/2 & 4 \\ 0 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{or} \quad A_G = \begin{bmatrix} \text{Inf} & 3 & \text{Inf} & \text{Inf} \\ 1 & \text{Inf} & 1/2 & 4 \\ \text{Inf} & \text{Inf} & \text{Inf} & 7 \\ \text{Inf} & \text{Inf} & \text{Inf} & \text{Inf} \end{bmatrix}$$

"length of path" becomes "total weight of path"

(ex)

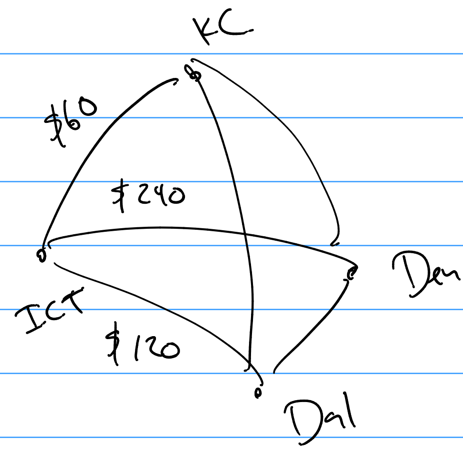


a, b, c, d

length = 10.5

Making weighted graphs:

(ex) (1) cost



(2) time

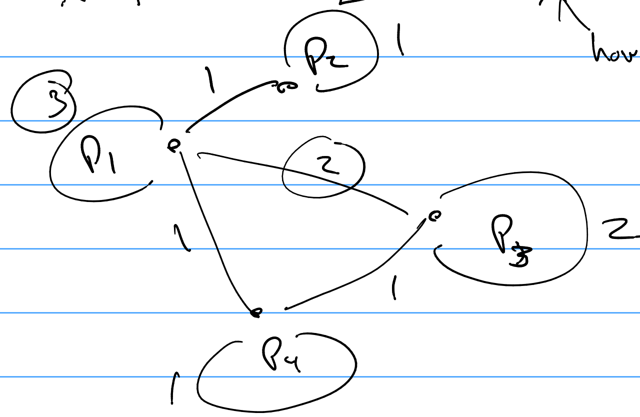
From incidence Matrix

$$\begin{array}{c|ccc} & C_1 & C_2 & C_3 \\ \hline P_1 & 1 & 1 & 1 \\ P_2 & 0 & 1 & 0 \\ P_3 & 1 & 0 & 1 \\ P_4 & 1 & 0 & 0 \end{array} = M_G$$

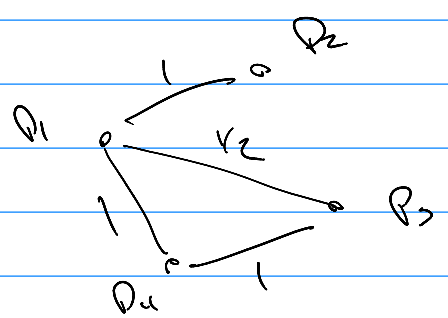
$$M_G M^T \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 2 & 1 \\ 1 & 1 & 0 & 0 \\ 2 & 0 & 2 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \leftarrow G$$

$P \times C \qquad C \times P$

how many clubs are you in



$$W_G = \begin{bmatrix} \text{Inf} & 1 & .5 & 1 \\ 1 & \text{Inf} & \text{Inf} & \text{Inf} \\ .5 & \text{Inf} & \text{Inf} & 1 \\ 1 & \text{Inf} & 1 & \text{Inf} \end{bmatrix}$$



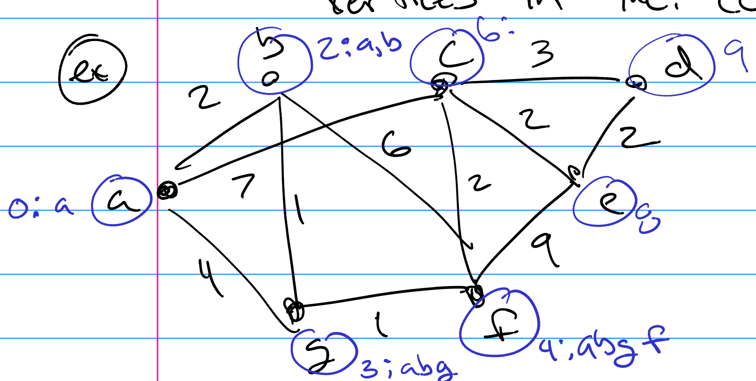
Path $P_1, P_2, P_1, P_4, P_3 = 4$

$P_1, P_3 = 1/2$

Uses ① shortest = lowest cost path from V_a to V_b

Dijkstra's Algorithm. this finds costs and paths of least cost from a given 1st vertex and then sorts all other vertices in inc. cost.

ex



Start @ a

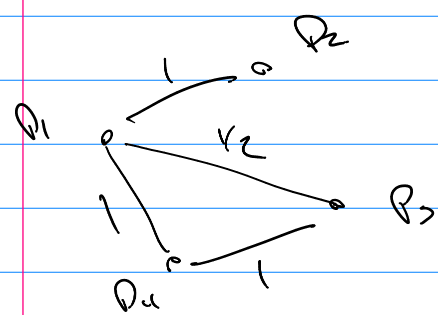
a : 0 : a
b : 2 : a,b
g : 3 : a,b,g
f : 4 : a,b,g,f
c : 6 : a,b,g,f c
e : 8

Scratch

a,c 7
b,f 8
g,f 4

→ dijkstra's is $\Theta(n^2)$ computational cost

$\boxed{\text{Use}}$ ex person x person example above...



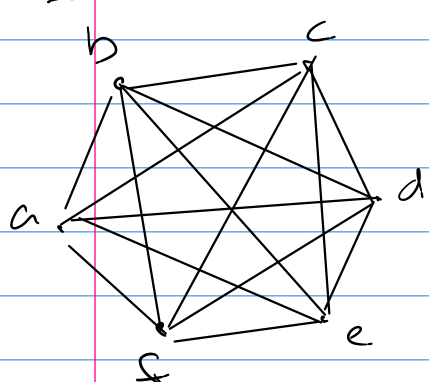
Application: ① run dijkstra for all vertices.

$p_1 \to$ table
$p_2 \to$ table
$p_3 \to$ table
$p_4 \to$ table

② for all tables how many times is some p_i in the paths of least cost?

③ sum all the times p_i is in a least path.

Traveling Salesman K_n weighted graph



$\deg(v) = 5 \geq \frac{6}{2}$ ① by Dirac's we have a hamilton circuit.

② $\boxed{\text{but}}$ b/c K_n has every vertex connected to every vertex then there are $(n-1)!$ Hamilton Circuits

K_6 $|V| = 6$ So K_6 has $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

$|E| = 15$

③ Note: if you say $a,b,c,d,e,f,a = a,f,e,d,c,b,a$

then $\frac{(n-1)!}{2}$ uniq hamilton circuits

So K_6 has $\frac{5!}{2} = 60$ $\boxed{\text{uniq}}$ circuits.

traveling Salesman problem is # of the $\frac{(n-1)!}{2}$ unique hamilton circuits in a K_n weighted graph $\Rightarrow$ find least cost.

b/c unig. and cost of one does not say anything about another hamilton circuit we must know all.

Approximation: $\left(\begin{matrix}\text{real}\\ \text{ans}\end{matrix}\right) \le \boxed{\text{Approx}} \le C\left(\begin{matrix}\text{real}\\ \text{ans}\end{matrix}\right)$

algorithm that gives a triangular boundry. in Polynomial time.

Problems and Computational Time