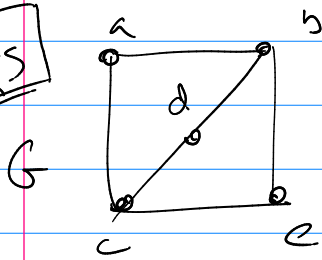


# Math 322

G's



$|V| = 5$

10.5 #35 Hamilton Circuit

check Dirac's:  $\deg(v) \geq \frac{|V|}{2}$

this graph: all  $v \quad \deg(v) \geq 3$

b/c  $\deg(a) = \deg(c) = \deg(e) = 2 \not\geq 3$

Dirac's says nothing

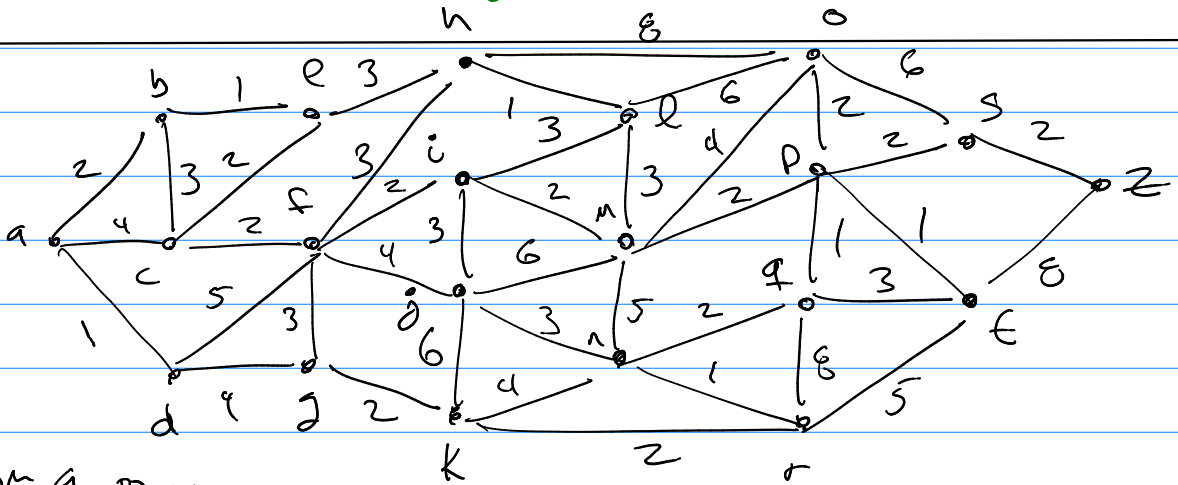
check Ore's:  $\deg(x) + \deg(y) \geq |V|$   $u, v$  are non-adj.

but  $\deg(a) + \deg(e) = 4 \not\geq 5$  so doesn't apply.

See video for rest of argument on why no Hamilton circuits.

10.6 #4

$|V| = 21$



Dijkstra's from a to ...

- (1) (1) a to a cost 0
- (2) (2) a to everywhere else (paths + costs)

(2) b to ...

(3) c to ...

(21) z to ...

all together we have  $21 \cdot 20 = 420$  paths

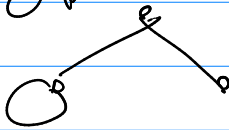
Exam

12 probs  $\rightarrow$  110pts = 100%

101/102 (3 probs)

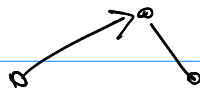
① Name those graphs

(ex) a)



pseudograph

b)



Mixed

c)



Simple directed

d)

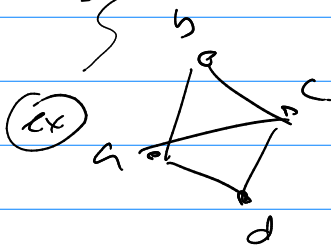


Directed multigraph

②

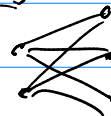
Given  $G=(V,E)$

state:  $|V|, |E|, \deg(v)$   
 $\deg^+(v), \deg^-(v)$



(ex)

$K_{2,2}$



(ex)  $Q_3$

etc


③

Given  $G_1, G_2$

- $\rightarrow$  Find
- a) subgraph induced by subset of  $V$
  - b)  $G_1 \cup G_2$
  - c) edge contraction.

103

Representing  $G=(V,E)$  and Isomorphisms (3 probs)

① Given  $G$  as  show  $A_G$

② show  $G_1, G_2$  are not isomorphic

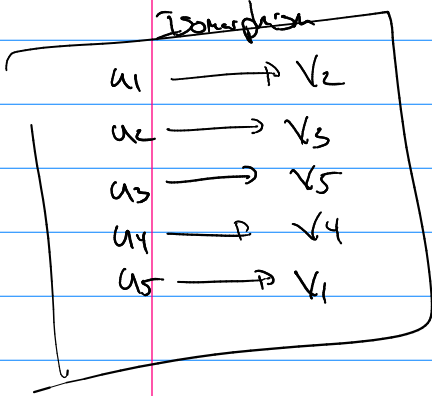
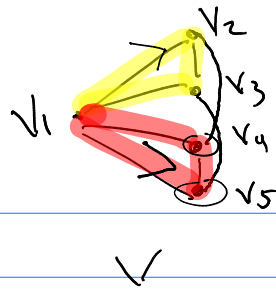
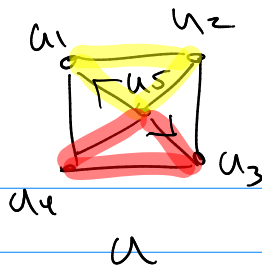
③ show  $G_1, G_2$  are isomorphic

a) find isomorphism

b)  $A_{G_1}, A_{G_2}$  in order of isomorphism:  $A_{G_1} = A_{G_2}$

ex

5 vertices



$$A_u = \begin{matrix} u_1 & \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \\ u_2 & \\ u_3 & \\ u_4 & \\ u_5 & \end{matrix}$$

$$A_v = \begin{matrix} v_2 & v_3 & v_5 & v_4 & v_1 \\ v_2 & \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \\ v_3 & \\ v_5 & \\ v_4 & \\ v_1 & \end{matrix}$$

### 104 Connectivity (2 probs)

① given  $G=(V,E)$   $\chi(G)$ ,  $\kappa(G)$ ,  $\min_{v \in V} \deg(v)$

② directed graph: strongly, weakly connected?

### 105 Euler / Hamilton Paths / Circuits (3 probs)

① state an Euler Circuit.

② bridge problem or cut puzzle

③ Hamilton Circuit problem

$\rightarrow$  state / use Dirac's

$\rightarrow$  state / use Ore's

$\rightarrow$  find or disprove Hamilton circuit.

### 106 Shortest Path (1 prob)

① Dijkstra's