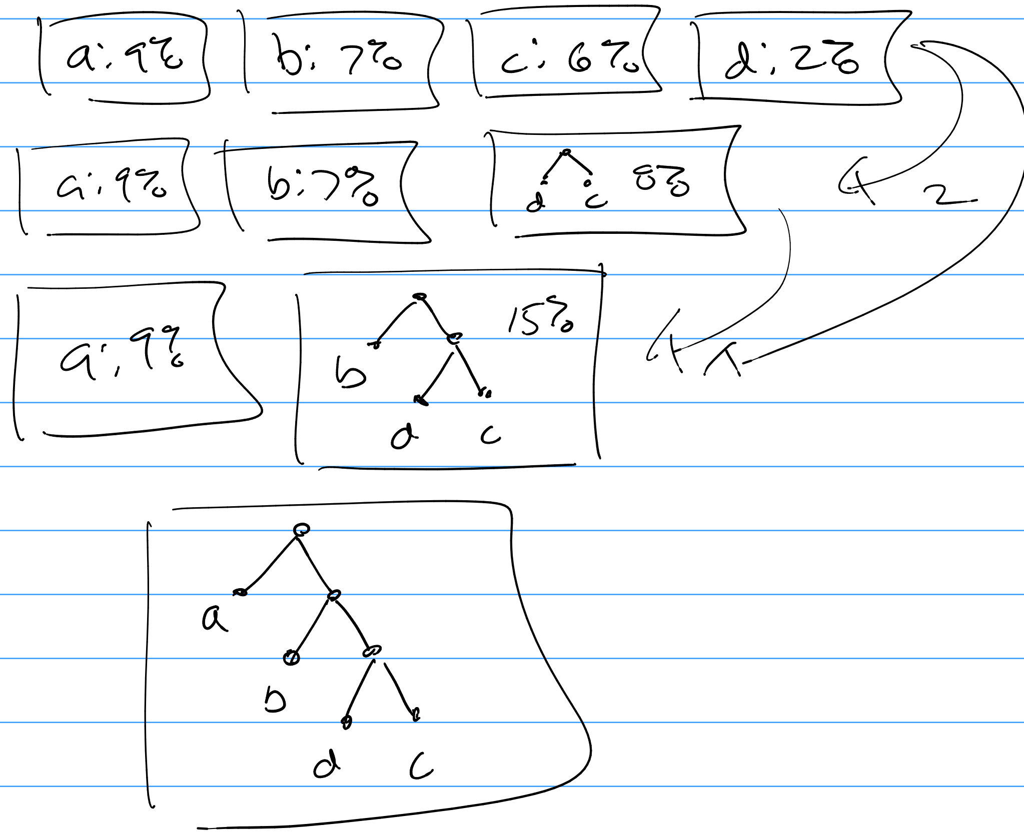
Math 322

Huffman:



$\boxed{12.2}$ $f : B^n \to B$ ex $f(x_1, x_2, .., x_n) =$ expression

table:

x_1	x_2 .. x_n	f
1	1 .. 1 1	1 → 0 or 1
1	1 .. 0 1	
⋮		
0	0 .. 0 0	1

2^n rows

total uniq. tables $(2)^{2^n}$

So there are only 2^{2^n} uniq $f(x_1, .., x_n)$

ex

x	y	z	f = (expression)?
1	1	1	1 ← $x \cdot y \cdot z$
1	1	0	0
1	0	1	1 ← $x \cdot \bar{y} \cdot z$
1	0	0	0
0	1	1	0
0	1	0	0
0	0	1	0
0	0	0	1 ← $\bar{x} \cdot \bar{y} \cdot \bar{z}$

$$f(x,y,z) = (x \cdot y \cdot z) + (x \cdot \bar{y} \cdot z) + (\bar{x} \cdot \bar{y} \cdot \bar{z})$$

Focus on min-term / max term.

X or $\overline{X}$

① **Min term:** product of all variables as literals

ex $(X_1 \cdot X_2 \cdot \overline{X_3} \cdot X_4 \cdot \ldots \cdot \overline{X_n})$

all X_i as X_i or $\overline{X_i}$

② **Maxterm:** Sum of all variables as literals

ex $(X_1 + X_2 + \overline{X_3} + \ldots + X_n)$

all X_i as X_i or $\overline{X_i}$

Why?:

① minterms are only 1 when all literals are 1's

ex $(X_1 \cdot X_2 \cdot \overline{X_3})$ gives 1 only when $X_1 = 1$

$X_2 = 1$

$\overline{X_3} = 1 \Rightarrow X_3 = 0$

② maxterms are only 0 when all literals are 0's

ex $(X_1 + \overline{X_2} + X_3)$ gives 0 only when $X_1 = 0$

$\overline{X_2} = 0 \rightarrow X_2 = 1$

$X_3 = 0$

So

X y z	assoc. min term (when 1?)	assoc max term (when 0?)
1 1 1	$X_1 \cdot X_2 \cdot X_3$	$(\overline{X_1} + \overline{X_2} + \overline{X_3})$
1 1 0	$X_1 \cdot X_2 \cdot \overline{X_3}$	$(\overline{X_1} + \overline{X_2} + X_3)$
1 0 1	$X_1 \cdot \overline{X_2} \cdot X_3$	$(\overline{X_1} + X_2 + \overline{X_3})$
1 0 0	$X_1 \cdot \overline{X_2} \cdot \overline{X_3}$	$(\overline{X_1} + X_2 + X_3)$
0 1 1	$\overline{X_1} \cdot X_2 \cdot X_3$	$(X_1 + \overline{X_2} + \overline{X_3})$
0 1 0	$\overline{X_1} \cdot X_2 \cdot \overline{X_3}$	$(X_1 + \overline{X_2} + X_3)$
0 0 1	$\overline{X_1} \cdot \overline{X_2} \cdot X_3$	$(X_1 + X_2 + \overline{X_3})$
0 0 0	$\overline{X_1} \cdot \overline{X_2} \cdot \overline{X_3}$	$(X_1 + X_2 + X_3)$

$\boxed{A}$ Minterm Expansion / Sum of Products / Disjunctive Normal Form

→ $\boxed{\text{Sum of minterms}}$

(ex)

x	y	f
1	1	0
1	0	1 ← $x\cdot\bar{y}$
0	1	0
0	0	1 ← $\bar{x}\cdot\bar{y}$

$$f(x,y) = (x\cdot\bar{y}) + (\bar{x}\cdot\bar{y})$$

x	y	f
1	1	0
1	0	1 ← $x\bar{y}$
0	1	0
0	0	0

$$f(x,y) = (x\cdot\bar{y})$$

Note:
$\neg(x \to y)$
$\neg(\neg x \vee y)$
$x \wedge \neg y$

$\boxed{B}$ Maxterm expansion / Product of Sums / Conjunctive Normal Form

→ $\boxed{\text{Product Maxterms}}$

x	y	f
1	1	0 ← $(\bar{x}+\bar{y})$
1	0	1
0	1	0 ← $(x+\bar{y})$
0	0	1

$$f(x,y) = (\bar{x}+\bar{y})(x+\bar{y})$$

x	y	f
1	1	1
1	0	0 ← $(\bar{x}+y)$
0	1	1
0	0	1

$$f(x,y) = (\bar{x}+y)$$

(ex)

focus on 1's ≡ minterm

x	y	z	f
1	1	1	0
1	1	0	1 ← $(x\cdot y\cdot\bar{z})$
1	0	1	0
1	0	0	0
0	1	1	1 ← $(\bar{x}\cdot y\cdot z)$
0	1	0	0
0	0	1	0
0	0	0	1 ← $(\bar{x}\cdot\bar{y}\cdot\bar{z})$

$$f(x,y,z) = (xy\bar{z}) + (\bar{x}yz) + (\bar{x}\bar{y}\bar{z})$$

focus on 0's ≡ use Maxterms

x	y	z	f	
1	1	1	0	$(\bar{x}+\bar{y}+\bar{z})$
1	1	0	1	
1	0	1	0	$(\bar{x}+y+\bar{z})$
1	0	0	0	$(\bar{x}+y+z)$
0	1	1	1	
0	1	0	0	$(x+\bar{y}+z)$
0	0	1	0	$(x+y+\bar{z})$
0	0	0	1	

$$f(x,y,z) = (\bar{x}+\bar{y}+\bar{z})(\bar{x}+y+\bar{z})(\bar{x}+y+z)(x+\bar{y}+z)(x+y+\bar{z})$$

Q How to go from expression into Sum of Minterms? Product of maxterms?

ex $f(x,y,z) = (xz) + (y)$

Tech #1 use a table

x	y	z	xz	$xz+y$
1	1	1	1	1
1	1	0	0	1
1	0	1	1	1
1	0	0	0	0
0	1	1	0	1
0	1	0	0	1
0	0	1	0	0
0	0	0	0	0

$$f(x,y,z) = (xyz)+(xy\bar{z})+(x\bar{y}z)+(\bar{x}yz)+(\bar{x}y\bar{z})$$
$$f(x,y,z) = (\bar{x}+y+z)(x+y+\bar{z})(x+y+z)$$

Tech #2 use $b \cdot 1 = b$ $\quad b + 0 = b$

and $(b+\bar{b}) = 1$ $\quad (b \cdot \bar{b}) = 0$

ex
$$f(x,y,z) = (xz) + (y) = (x \cdot 1 \cdot z) + (1 \cdot y \cdot 1)$$
$$= (x(y+\bar{y})z) + ((x+\bar{x})y)$$
$$= (xyz) + (x\bar{y}z) + (xy \cdot 1) + (\bar{x}y \cdot 1)$$
$$= (xyz) + (x\bar{y}z) + (xy(z+\bar{z})) + (\bar{x}y(z+\bar{z}))$$
$$= (xyz) + (x\bar{y}z) + (xyz) + (xy\bar{z}) + (\bar{x}yz) + (\bar{x}y\bar{z})$$
$$= (xyz) + (x\bar{y}z) + (xy\bar{z}) + (\bar{x}yz) + (\bar{x}y\bar{z})$$

$b \wedge 1 = b \qquad b \vee 0 = b$

$b \vee \bar{b} = 1 \qquad b \wedge \bar{b} = 0$

$f(x,y,z) = xz + y$

$= (x \wedge z) \vee y$ prod of Maxterm $(\ \vee\ \vee\ ) \wedge (\ \vee\ \vee\ )$

$= (x \vee y) \wedge (y \vee z)$

$= (x \vee y \vee \overset{z \wedge \bar{z}}{0}) \wedge (\overset{x \wedge \bar{x}}{0} \vee y \vee z)$

$= ((x \vee y \vee)(z \wedge \bar{z})) \wedge ((x \wedge \bar{x}) \vee y \vee z))$

$= (x \vee y \vee z) \wedge (x \vee y \vee \bar{z}) \wedge (x \vee y \vee z) \wedge (\bar{x} \vee y \vee z)$

$= (x \vee y \vee z) \wedge (x \vee y \vee \bar{z}) \wedge (\bar{x} \vee y \vee z)$

vs $f(x,y,z) = xz + y = (x \cdot z) + y$

$= (x+y) \cdot (z+y)$

$= (x+y+0) \cdot (0+y+z)$

$= ((x+y+)(z \cdot \bar{z})) \cdot ((x \cdot \bar{x}) + y + z)$

$=$ etc.

$\boxed{\text{Note:}}$ $f : B^n \to B \qquad f(x_1, x_2, \ldots, x_n) = \underline{\text{expression}}$

by our work with Maxterm / minterm expansions we know that $\boxed{\text{all}}$ 2^{2^n} unique functions can be written using

<u>only</u> $x_1, x_2, \ldots, x_n, +, \cdot, \bar{\ }$

call $\{+, \cdot, \bar{\ }\}$ to be functionally complete.

$\boxed{\text{Q}}$ Can we have fewer?

by DeMorgans: $\overline{X+y} = \bar{X}\cdot\bar{y}$ so $X+y = \overline{\bar{x}\cdot\bar{y}}$

So if you had (ex) $f(x,y) = (\bar{X}+y) = \boxed{\overline{\bar{\bar{x}}\cdot\bar{y}}}$

b/c $\square + \triangle = \overline{\bar{\square}\cdot\bar{\triangle}}$ $\{\cdot, \bar{\ }\}$ is functionally complete

Also

by DeMorgans $\overline{X\cdot y} = \bar{X}+\bar{y} \rightarrow X\cdot y = \overline{\bar{X}+\bar{y}}$

b/c $\square\cdot\triangle = \overline{\bar{\square}+\bar{\triangle}}$ $\{+, \bar{\ }\}$ is functionally complete

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Consider

x	y	$x \downarrow y$	$x \mid y$
1	1	0	0
1	0	0	1
0	1	0	1
0	0	1	1

not (or) — not(And)

NOR — NAND

$x \downarrow x = \bar{x}$

$(x\downarrow y)\downarrow(x\downarrow y) = x+y$ so $\{\downarrow\}$ is functionally complete

$(x\downarrow x)\downarrow(y\downarrow y) = x\cdot y$

$x \mid x = \bar{x}$

$(x\mid y)\mid(x\mid y) = x\cdot y$ So $\{\mid\}$ is functionally complete.

$(x\mid x)\mid(y\mid y) = x+y$