Math 322

Turing Machines

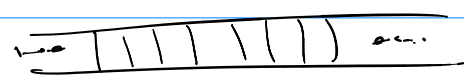
$$f : S \times I \to S \times I \times \{R, L\}$$

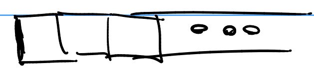
Deterministic given $(\boxed{s, i}, s, i, m)$

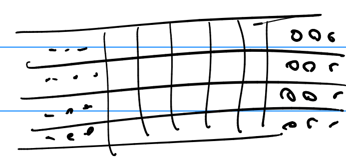
given any input pair we have one output

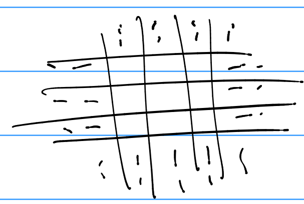
Non-Det given $(\boxed{s, i}, s, i, m)$

given any input pair we can have one or more outputs.

Memory Mods — typical  infinite tape

—  infinite on only one side

— multi tape 

— 2D 

but none of the above add or remove from phrase-structure grammar generated sets.

→ So the normal $f : S \times I \to S \times I \times \{\text{right}, \text{left}\}$ on infinite tape memory covers everything that can be done

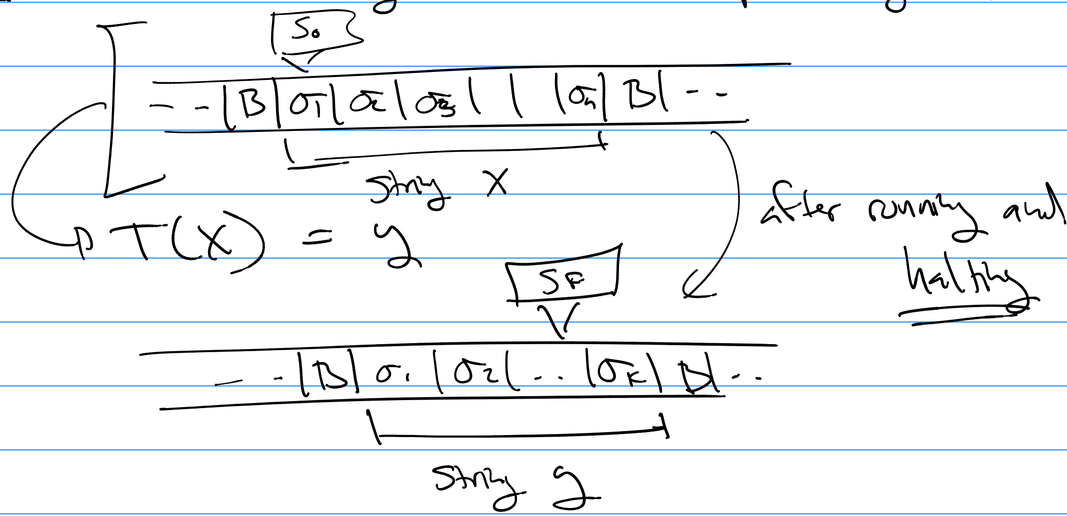
Church-turing thesis : If a problem has an effective algorithm then a turing machine exists that solves the problem.

Solvable / Decidable / Computable

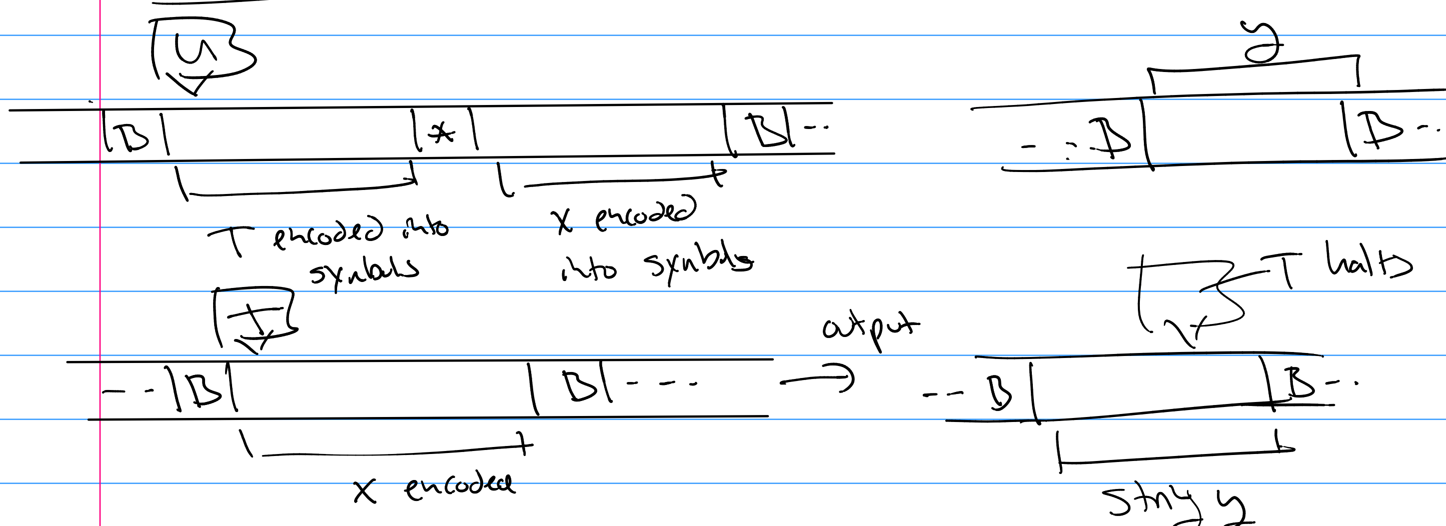
→ Yes/no problems

Are any problems unsolvable?

Background: T is a turing machine with input string of X

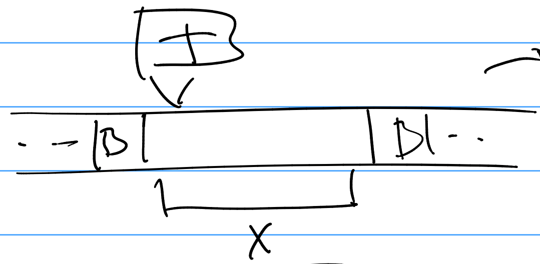


Universal Turing Machine $U(T, X) = y$

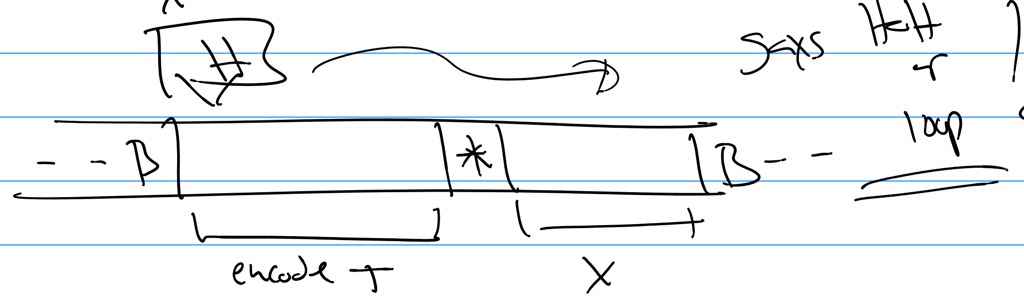


$T(X) = y$

Q does $H(T, X)$ exist such that



→ if $T(x)$ loops $H(T,x)$ says loop
$T(x)$ halts $H(T,x)$ says halt



Thm given T and input string X there is $\boxed{\text{no}}$ turing machine that can determine whether $T(x)$ halts or loops.

Pf assume $H(T,X)$ is the halting turing machine.
so if $T(x)$ halts $H(T,X)$ said halt
if $T(x)$ loops $H(T,X)$ said loop.

consider the flipper machine... $F(T)$

$F(T)$ will say halt if $\boxed{H(T,T)}$ says loop. $T(T)$

if $T(T)$ would loop $H(T,T)$ says loop
if $T(T)$ would halt $H(T,T)$ says halt.

Question: $F(F)$

Case 1: $F(F)$ says loop
$H(F,F)$ says halt $F(F)$ halted
but $F(F)$ say loop } contradiction

case 2 $H(P,P)$ says loop means $P(P)$ loop } contradiction
but $P(P)$ says halt

So H can not exist.

Computable functions: (find a turing machine to compute $f(n_1, n_2, \dots, n_k)$)

ex $f(n) = n \bmod 3$ $T = ?$ find 5-tuples

$f(0)=0$	$1 \xrightarrow{T} 1$
$f(1)=1$	$11 \xrightarrow{T} 11$
$f(2)=2$	$111 \xrightarrow{T} 111$
$f(3)=0$	$1111 \xrightarrow{T} 1$
$f(4)=1$	$11111 \to 11$
$f(5)=2$	$111111 \to 111$
$f(6)=0$	$1111111 \to 1$

S_0
7 -- B 1 1 1 1 1 1 1 1 B --
S_0
4 -- B B B B 1 1 1 1 1 B --
S_0
1 ... B B B B B B 1 1 B --
S_{11}
... B B ... B B B B --
S_1
- - B 1 --
S_F
B 1 1

$(S_0, 1, S_1, B, R)$
$(S_1, 1, S_{11}, B, R)$
$(S_{11}, 1, S_0, B, R)$
$(S_0, B, S_{11}, 1, L)$
$(S_{11}, B, S_1, 1, L)$
$(S_1, B, S_F, 1, L)$

S_F
-- 1 B 1 B 1 1 1 --
S_F S_{11} S_1 S_0
-- B | B | [illegible] | [illegible] | B

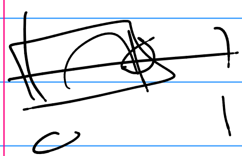
$|\text{all functions } f : \mathbb{Z}^+ \to \{0, 1, \dots, 9\}| \Rightarrow$ <u>Uncountable</u>

from $\mathbb{R}$ being uncountable

we can consider any real between 0 and 1 to a function

$\to$ $f_n = 0.1212212221222221\dots$

$f_n(n) = d_n$

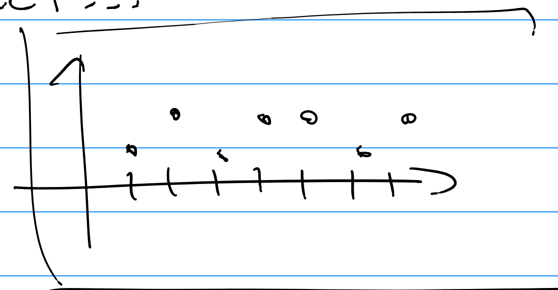


$f_n(1) = 1$

$f_n(2) = 2$

$f_n(3) = 1$

$f_n(4) = 2$



But, effective algorithm / turing machines are countably infinite.