Math 322

!! Final Exam → Tues @ 3pm !!

4 Exams:

1 Exam → 4 probs
2 Exam → 4 probs
3 Exam → 4 probs
4 Exam → 4 probs

@ 10pts

150pts = 100%

Note:

← will be on test
exact? maybe
variation? maybe
concept? maybe

~~#~~ ← not on test

? # ← study! May be on test

Exam 1

1) Is the relation R consisting of all ordered pairs (a, b) such that a and b are people and have one common parent: reflexive, irreflexive, symmetric, antisymmetric, asymmetric, and/or transitive? If a property doesn't hold give a counter-example and **state the logical definitions of the properties** as you consider them.

?

2) Given the relation $R_1 = \{(a, b) | b = 2a\}$ and $R_2 = \{(a, b) | b = 3a - 1\}$ on the set of positive integers from 1 to 12. Give the list of ordered pairs for R_1 and R_2 and find the relation $R_1 \cap R_2$.

(understand R = "list" or digraph or matrix)

?

3) Represent the relation $R = \{(a, a), (a, c), (b, a), (c, a), (c, b)\}$ on the set $A = \{a, b, c, d\}$ as a digraph and a matrix.

4) Prove: ~~If~~ R on set A is transitive, ~~then~~ $\Leftrightarrow$ $\forall n R^n \subseteq R$, $n = 1, 2, 3, \ldots$.

?

5) For the set $A = \{a, b, c\}$, relation $R_1 = \{(a, a), (a, c), (b, b), (c, a)\}$, and relation $R_2 = \{(a, b), (b, a), (b, b), (c, c)\}$. Represent the relations as matrices and then use matrix operations to find $R_1 \circ R_2$.

(understand operations using Matrices)

?

6) For the set $A = \{a, b, c\}$, relation $R_1 = \{(a, a), (a, c), (b, b), (c, a)\}$, and relation $R_2 = \{(a, b), (b, a), (b, b), (c, c)\}$. Represent the relations as matrices and then use matrix operations to find $R_1 \cap R_2$.

7) For $R = \{(a, a), (a, c), (a, d), (b, a), (b, d), (c, a), (c, d), (d, a), (d, c)\}$ on the set $A = \{a, b, c, d\}$ find the ...

a) Reflexive Closure as a matrix.

b) Symmetric Closure as a matrix.

c) trans. closure (know Warshall's ($M_{R^*} = M_R \vee M_R^{[2]} \vee \cdots \vee M_R^{[n]}$))

8) For $R = \{(a, a), (a, b), (a, d), (b, a), (b, c), (c, d), (d, c)\}$ on the set $A = \{a, b, c, d\}$ find the transitive closure using Warshall's Algorithm.

9) For $R = \{(a, a), (a, b), (b, a), (b, c), (c, a)\}$ on the set $A = \{a, b, c\}$ find the transitive closure using the join of powers of M_R.

?

10) Show that the relation R consisting of all pairs (f, g) such that the first derivative of f and the first derivative of g are equal is an equivalence relation on the set of all polynomials with real-valued coefficients.

11) For the relation given above which functions are in the same equivalence class as $f(x) = 2x - 1$?

?

12) Show that $(\because^{+}, |)$ a partial ordering.

13) For the given Hasse diagram ...

a) State the maximal, minimal, greatest, and least elements.

b) Create a topological sort. (Note: always take the right minimal first)

Exam 2 (graph theory)

1a) Name the graph.

1b) Name the graph.

1c) Name the graph.

Name?

Undirected: simple, multi, pseudo | directed: simple directed, directed multi.

mixed

Know: C_n, K_n, W_n, Q_n, $K_{m,n}$, bipartite

2) Draw the graph W_5, label each vertex, and state the number of vertices, edges, and degree for each vertex.

3) For the two given graphs G_1 and G_2 find a) the subgraph of G_1 induced by taking a, b, c. And b) find the union of G_1 and G_2.

4) Find the adjacency matrix for the given graph.

5) Why are the graphs not isomorphic?

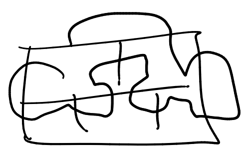
6) Verify that the graphs are isomorphic.

7) Draw a directed multi-graph with 5 vertices that is weakly connected and not strongly connected. State why it is not strongly connected.

or undirected graph know "connected"

8) For the given undirected graph find $\kappa(G)$ and $\lambda(G)$. State the vertices that make a minimal vertex cut. State the edges that make a minimal edge cut.

9) Find an euler circuit in the given graph.



10) For the given cut puzzle, is there a continuous curve that will cross each of the line segments exactly once? Justify your answer.

or any application of euler circuits or euler paths.

11) Can Dirac's and/or Ore's Theorems be applied to Q_3? (Explain why or why not) Find a Hamilton Circuit for Q_3.

state them

12) Find the paths and lengths for the shortest paths between b and every other vertex in the graph.

$n = i + l$ $\quad h \geq \lceil \log_m l \rceil$

$n = mi + 1$

EXAM 3 Trees / Game

1) You receive the following message via some social media application "Send the message 'I love to count in an advanced way' to 4 of your friends and you will get an A in Math 322!" If a total of 100 people send the message before it stops, how many people are in the tree? How many edges are in the tree? How many received it and did not send it out? What could you say about the height of the tree?

2) Prove: For an $m-$ary tree of height h with l leaves, $l \leq m^h$.

3) In a best case situation, how many weighings of a balance scale are needed if given four coins you may have a heavy counterfeit? Construct a decision tree to find the counterfeit or determine if there is no counterfeit.

4) Create a binary search tree for "mark, joe, adam, bat, kim, silly, cat, hat".

5) Draw the game tree for tic-tac-toe for the given starting position. Player "x" is the one with the next move. Who wins the game if both players follow an optimal strategy?

6) Create the Huffman Code tree if a:25%, e:19%, i:18%, t:17%, s:14%, d:7% and encode "sad"

7) Write the inorder, preorder, and postorder traversal of the given tree.

8) For the standard expression $\sin[(2x + x^2)/(x + 1)]$

a) Construct the rooted tree for the given expression.

b) Write the expression using post-fix notation.

c) Write the expression using pre-fix notation.

9) Use a bit table to verify De Morgan's laws $\overline{x + y} = \bar{x} \cdot \bar{y}$.

10) Using only the Identity, Complement, Associative, Communtative, and/or Distributive laws of a Boolean Algebra verify that $x + x = x$.

11) Find the sum of products for $F(x, y, z) = x \cdot (x + (y \cdot z))$ ~~without~~ using a table.

(only one)

12) Find the product of sums for $F(x, y, z) = (x + y) \cdot z$ by using a table.

EXAM 4

1) For the grammar with $V = \{0, 1, A, B, S\}$, $T = \{0, 1\}$, and the productions $S \to 0A$, $S \to B1$, $S \to \lambda$, $A \to 0B1$, $B \to 1$, and $A \to 0$ find L(G).

2) Name the grammar type (just give its type number) and circle the productions that prevent it from being the next type.

a) $S \to A, S \to B, S \to \lambda, A \to Sb, B \to aB, A \to a$, and $B \to b$

b) $S \to AB, A \to aA, B \to bB, A \to a$, and $B \to b$

c) $S \to ASB, S \to \lambda, B \to aAb, A \to a$, and $A \to B$

d) $S \to \lambda, S \to aA, A \to bB, B \to a$, and $A \to b$

3) Construct a finite-state machine with output that models a candy machine that accepts only pennies. Candy costs 2 cents and the machine returns the money for any amount greater than 2 cents. The customer can push buttons to receive candy or to return pennies. Represent the machine with a state table and state diagram.

4) Construct a finite-state machine with output that delays input by two bits using 00 for the delay. Represent the machine with a state diagram.

5) Determine the language recognized by a given deterministic finite-state automaton.

(only one is used)

6) Determine the language recognized by a given non-deterministic finite-state automaton.

7) Construct a deterministic finite-state automaton that recognizes the same language as the given non-deterministic finite-state automaton.

8) Using the constructions described in the proof of Kleene's Theorem, find a non-deterministic finite-state automaton that recognizes $1 \cup (10)^*$.

9) Construct a non-deterministic finite-state automaton that recognizes the language generated by the regular grammar with $V = \{0, 1, A, S\}$, $T = \{0, 1\}$, and the productions $S \to 0A$, $S \to 1B$, $S \to \lambda$, $A \to 1A$, $A \to 1$, $B \to 0B$, $B \to 1$.

10) Let T be the Turing machine defined by the five-tuples: $(s_0, 0, s_1, 0, R)$, $(s_0, 1, s_1, 0, L)$, $(s_0, B, s_1, 1, R)$, $(s_1, 0, s_2, 1, R)$, $(s_1, 1, s_2, 1, R)$, and $(s_1, B, s_2, 1, R)$. Run the Turning machine on the below initial tape, write each of the positions, and determine the tape when T halts. Does T recognize the input string? What is T's task?

Inital Tape: ... B,B,B,0,1,1,0,1,0,B,B,B, ...

11) Construct a Turing machine for the non-negative integers in unary format that computes the function $f(n) = n + 3$. Run your machine on the input 1,1,1,1.

B 1 1 B B

12) Construct a Turing machine for $f(n) = n \bmod 3$. Run your machine on the input: 1,1,1,1,1,1.

11, 12, "13"

$f(n) = n + c$

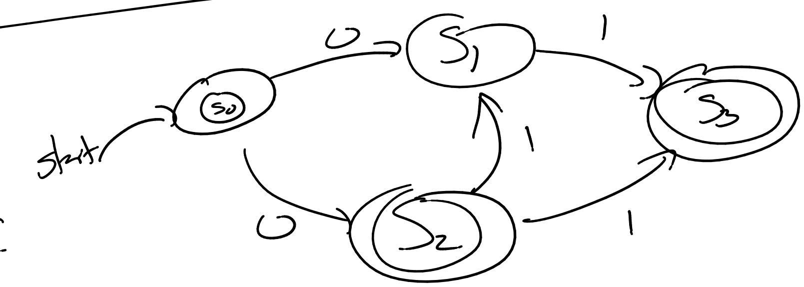
$f(n) = n \bmod m$

$f(n_1, n_2) = n_1 + n_2$

One will be on final.

ex

Non-det



det: