

Math 511

Linear Algebra

Math = toys ⊕ roles
↳ Problem Solving

Elem. Algebra

Solve

$$\begin{array}{r} 2x + 4 = 3 \\ -4 \quad -4 \\ \hline 3x = 2 \end{array}$$

$$\begin{aligned} x &= \frac{2}{3} \\ x &= 2/3 \end{aligned}$$

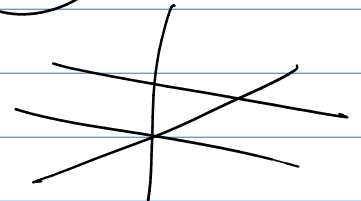
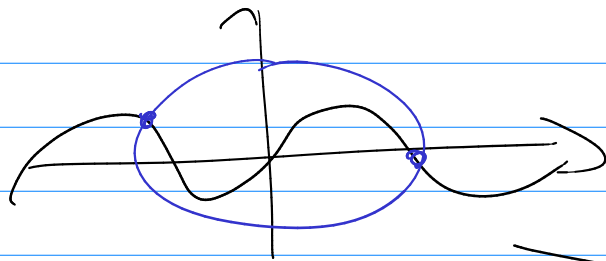
$$\begin{aligned} \left(\frac{1}{3} \cdot 3\right)x &= \frac{1}{3} \cdot 2 \\ 1 \cdot x &= \frac{2}{3} \end{aligned}$$

Sys. of Equations

linear

$y = \sin x$

$x^2 + y^2 = 4$



System of linear eqn's

a_{ij}, b_i are real numbers
(constants)

$n \times n$

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \dots + a_{2n}x_n = b_2$$

⋮

$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m$$

m -eqn's (rows)

n -variables (col's)

(ex)
$$\begin{cases} 3x_1 + 2x_2 - x_3 = 4 \\ x_1 - \pi x_2 + e x_3 = \sqrt{2} \end{cases} \quad \underline{2 \times 3 \text{ system}}$$

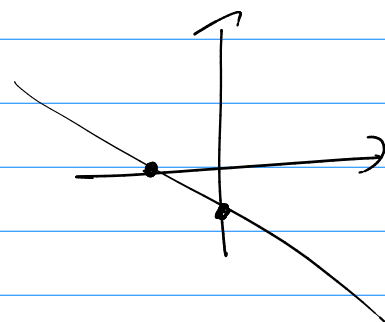
$$\begin{matrix} a_{11} = 3 & a_{12} = 2 & a_{13} = -1 & b_1 = 4 \\ a_{21} = 1 & a_{22} = -\pi & a_{23} = e & b_2 = \sqrt{2} \end{matrix}$$

Var's x_1, x_2, x_3

Solve → what makes each equality true?

(ex) $\begin{cases} x = 3/2 \end{cases}$ solk. $\boxed{3/2}$

(ex) $\begin{cases} x + y = -1 \end{cases}$ solk. $\begin{matrix} (-1, 0) \\ (2, -3) \end{matrix}$



(ex) $n \times n$ system (square systems)

tech to solve:

① substitution

(ex) $\boxed{(2, -5)}$

$$\begin{cases} 3x + y = 1 \\ x - y = 7 \end{cases}$$

$$\begin{aligned} & \rightarrow y = x - 7 \\ \text{Sub: } & 3x + (x - 7) = 1 \\ & 4x = 8 \\ & \boxed{x = 2} \\ & \boxed{y = -5} \end{aligned}$$

② elimination:

$$\begin{array}{r} 3x + y = 1 \\ x - y = 7 \\ \hline 4x = 8 \end{array} \rightarrow \begin{cases} 3x + y = 1 \\ (4x = 8) / 4 \end{cases}$$

$$\begin{cases} 3x + y = 1 \\ x = 2 \end{cases}$$

$$\begin{aligned} 3(2) + y &= 1 \\ \underline{y} &= -5 \end{aligned}$$

$$(2, -5)$$

$$\begin{cases} 3x + y = 1 \\ x - y = 7 \end{cases} \xrightarrow[\text{position}]{\text{keep}} \left[\begin{array}{cc|c} 3 & 1 & 1 \\ 1 & -1 & 7 \end{array} \right]$$

$$\left[\begin{array}{cc} 3 & 1 \\ 1 & -1 \end{array} \right] \leftarrow \text{coef. matrix}$$

augmented matrix

$$\begin{cases} 3x + y = 1 \\ x - y = 7 \end{cases} \quad (\times 5) \quad \left[\begin{array}{cc|c} 3 & 1 & 1 \\ 1 & -1 & 7 \end{array} \right]$$

row 1 + row 2 = new row 2

$$\begin{cases} 3x + y = 1 \\ 4x = 8 \end{cases} \quad \left[\begin{array}{cc|c} 3 & 1 & 1 \\ 4 & 0 & 8 \end{array} \right]$$

row 2 $\times \frac{1}{4}$ = new row 2

$$\begin{cases} 3x + y = 1 \\ x = 2 \end{cases} \quad \left[\begin{array}{cc|c} 3 & 1 & 1 \\ 1 & 0 & 2 \end{array} \right]$$