Math 511

Q's 2.3 $\operatorname{adj}(A) = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ \vdots & & & \\ A_{n1} & - & - & A_{nn} \end{bmatrix}^T$

so $\boxed{A^{-1} = \frac{1}{\det(A)} \operatorname{adj}(A)}$

thm 2.3.1 A^{-1} exists let $b \in \mathbb{R}^n$

(Solve: ? $Ax = b$) def A_i to be A with a_i replaced by b.

$x = A^{-1} b$

ex $A_1 = [b \; a_2 \; a_3 \cdots a_n]$

$A_2 = [a_1 \; b \; a_3 \cdots a_n]$

etc

Soln: $x = A^{-1} b$ $\Rightarrow$ $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} \frac{\det(A_1)}{\det(A)} = x_1 \\ \frac{\det(A_2)}{\det(A)} = x_2 \\ \vdots \end{bmatrix}$

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to do 2.3.2c
$$\begin{cases} 2x_1 + x_2 - 3x_3 = 0 \\ 4x_1 + 5x_2 + x_3 = 8 \\ -2x_1 - x_2 + 4x_3 = 2 \end{cases}$$

what you "should" be able to do ... ① substitution ② elimination ③ aug. matrix + gauss/jordan

$$\begin{cases} 2x_1 + x_2 - 3x_3 = 0 \\ 4x_1 + 5x_2 + x_3 = 8 \\ -2x_1 - x_2 + 4x_3 = 2 \end{cases}$$

④ Matrix Algebra

$$A = \begin{bmatrix} 2 & 1 & -3 \\ 4 & 5 & 1 \\ -2 & -1 & 4 \end{bmatrix} \quad \mathbb{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \mathbb{b} = \begin{bmatrix} 0 \\ 8 \\ 2 \end{bmatrix}$$

System $\Rightarrow A\mathbb{X} = \mathbb{b}$

so $\mathbb{X} = A^{-1}\mathbb{b}$

to find A^{-1}

$$\left[\begin{array}{ccc|ccc} 2 & 1 & -3 & 1 & 0 & 0 \\ 4 & 5 & 1 & 0 & 1 & 0 \\ -2 & -1 & 4 & 0 & 0 & 1 \end{array}\right]$$

row ops

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & A^{-1} \\ 0 & 0 & 1 & \end{array}\right]$$

⑤ <u>Cramers</u> $\mathbb{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ for above..

$$A = \begin{bmatrix} 2 & 1 & -3 \\ 4 & 5 & 1 \\ -2 & -1 & 4 \end{bmatrix} \quad \mathbb{b} = \begin{bmatrix} 0 \\ 8 \\ 2 \end{bmatrix} \quad A_1 = \begin{bmatrix} 0 & 1 & -3 \\ 8 & 5 & 1 \\ 2 & -1 & 4 \end{bmatrix} \quad A_2 = \begin{bmatrix} 2 & 0 & -3 \\ 4 & 8 & 1 \\ -2 & 2 & 4 \end{bmatrix} \quad A_3 = \begin{bmatrix} 2 & 1 & 0 \\ 4 & 5 & 8 \\ -2 & -1 & 2 \end{bmatrix}$$

$\Rightarrow \det(A) = ?$ $\det(A_1) = ?$ $\det(A_2) = ?$ $\det(A_3) = ?$

$$\mathbb{X} = \begin{bmatrix} \det(A_1)/\det(A) \\ \det(A_2)/\det(A) \\ \det(A_3)/\det(A) \end{bmatrix}$$

Ex

stay with <u>type 3</u>

$$\begin{vmatrix} 2 & 1 & -3 \\ 4 & 5 & 1 \\ -2 & -1 & 4 \end{vmatrix} = \begin{vmatrix} 2 & 1 & -3 \\ 0 & 3 & 7 \\ 0 & 0 & 1 \end{vmatrix} \begin{matrix} \leftarrow \text{row}_2 + (-2)\text{row}_1 = N \text{ row}_2 \\ \leftarrow \text{row}_1 + \text{row}_3 = N \text{ row}_3 \end{matrix}$$

$= 6$

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add type 2

type 3

$$\begin{vmatrix} 2 & 1 & -3 \\ 4 & 5 & 1 \\ -2 & -1 & 4 \end{vmatrix} = \frac{1}{1/2} \begin{vmatrix} 1 & 1/2 & -3/2 \\ 4 & 5 & 1 \\ -2 & -1 & 4 \end{vmatrix}$$

$\det(E_{type2} A) = \det(B)$

$\alpha \det(A) = \det(B)$

$\det(A) = \frac{1}{\alpha} \det(B)$

$\boxed{\text{Exam}}$ 12 probs @ 10 pts each

110 pts = 100%

$\boxed{1.1}$ Systems of Eqn's (1 prob)

① Solve a system $\boxed{\text{without}}$ matrices.

ex

$$\begin{cases} 2x_1 + x_2 - 3x_3 = 0 \\ 4x_1 + 5x_2 + x_3 = 8 \\ -2x_1 - x_2 + 4x_3 = 2 \end{cases}$$

by substitution

$\to 2x_1 + x_2 - 3x_3 = 0$

$x_2 = 3x_3 - 2x_1$

$$\begin{cases} 4x_1 + 5(3x_3 - 2x_1) + x_3 = 8 \\ -2x_1 - (3x_3 - 2x_1) + 4x_3 = 2 \end{cases}$$

$$\begin{cases} -6x_1 + 16x_3 = 8 \\ x_3 = 2 \end{cases}$$

(now backsolve)

$\boxed{1.2}$ Systems by Augmented Matrices (2 probs)

① Solve System by Aug. Matrices (Gauss

$$\begin{cases} 2x_1 + x_2 - 3x_3 = 0 \\ 4x_1 + 5x_2 + x_3 = 8 \\ -2x_1 - x_2 + 4x_3 = 2 \end{cases}$$

$$\left[\begin{array}{ccc|c} 2 & 1 & -3 & 0 \\ 4 & 5 & 1 & 8 \\ -2 & -1 & 4 & 2 \end{array}\right]$$

$r_2 + (-2)r_1 = r_2$

$r_3 + r_1 = r_3$

$$\left[\begin{array}{ccc|c} 2 & 1 & -3 & 0 \\ 0 & 3 & 7 & 8 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$r_1 = \frac{1}{2} r_1$

$r_2 = \frac{1}{3} r_2$

$$\left[\begin{array}{ccc|c} 1 & 1/2 & -3/2 & 0 \\ 0 & 1 & 7/3 & 8/3 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$\to x_2 + \frac{7}{3}\underset{2}{\cancel{x_3}} = 8/3$

(now try backsolve) $x_3 = 2$ $x_2 = -2$ $x_1 = 4$

2nd prob of 1.2

② word problem → sys. of linear eqn's → Solve by aug. matrix

traffic (like application #1 p. 17)
or econ (like #4 p. 21-22)

Note: both had free variables.

1.3/1.4 Matrix/Vector Ops. and Algebra (4 probs)

① "Bunch" of matrix arith.
② Verify an inverse
③ p. 59 #14
④ Matrix Eqn → solve for a specific matrix.

$$AX + B = X + C \quad \text{solve for } C$$
$$AX - X = C - B$$
$$AX - I \cdot X = C - B$$
$$(A-I)X = C - B$$
$$\boxed{(A-I)X + B = C}$$

given $A = [\ \ ]$
$X = [\ \ ]$
$B = [\ \ ]$

1.5 Elementary Matrices (3 probs)

① find A^{-1} by $[A \mid I] \xrightarrow[\text{ops}]{\text{row}} [I \mid A^{-1}]$

② $A = LU$ each row op $= E_{\text{type } 1, 2 \text{ or } 3}$

③ - state thm 1.3.1
- state thm 1.5.2
- ex if $c_1 a_{11} + c_2 a_{12} + \cdots + c_n a_{1n} = 0$ for non-all zero c_i
  what does that mean for $A\mathbf{x} = 0$?
  what does it mean for A^{-1}?

$\boxed{1.6}$ Partitioned Matrices (0 prob)

$\boxed{2.1/2.2/2.3}$ det(A) (2 probs)

1st: understand thm 2.2.2

① det(A) by co-factors, by elimination.

② is A singular or non-singular?

ex $A = \begin{bmatrix} (1-x) & 2 & 3 \\ 0 & (x-1) & 1 \\ 0 & 0 & (x+2) \end{bmatrix}$

$\det(A) = (1-x)(x-1)(x+2)$

if $x = 1, -2$ the A^{-1} does not exist (singular)