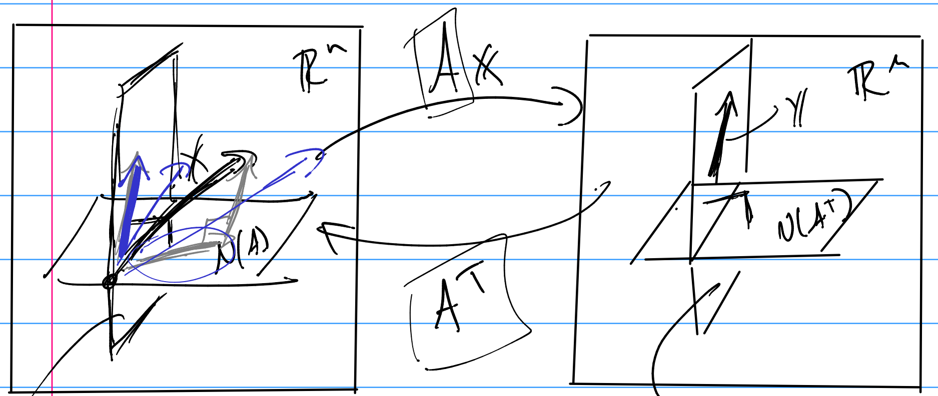
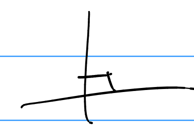
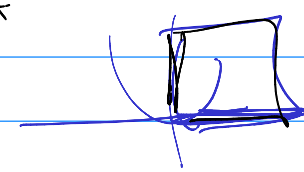
Math 511

Exam: thurs you get exam back
due Tues → Fix exam for 50% missed.



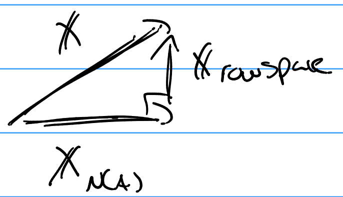
$$Ax = y$$
$m \times n$ $n \times 1$ $m \times 1$





row space of A = col. space of A^T

col. space of A = row space of A^T
= range of A

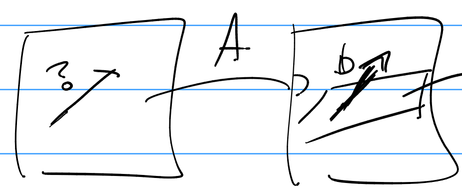


$$x = x_{N(A)} + x_{rowspace}$$

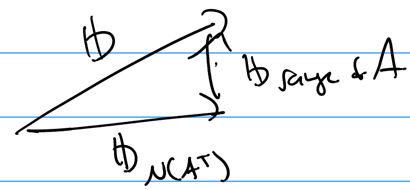
(1) $A(x) = A(\underline{x_{N(A)}} + x_{rowspace}) = 0_{\mathbb{R}^m} + A x_{rowspace}$

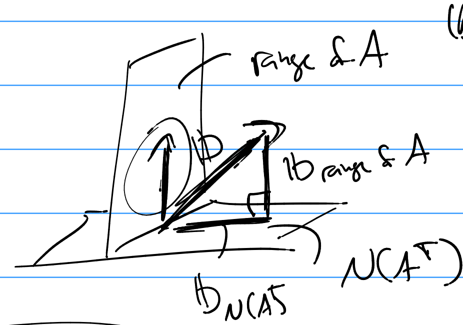
$$Ax = A\, x_{rowspace} = y$$

(2) Solve $Ax = b$
$b \notin$ col. space
→ so no soln.



range of A
= col. space of A
$b \notin$ range of A





Notice: $A^T b = A^T(b_{N(A^T)} + b_{\text{range of } A})$

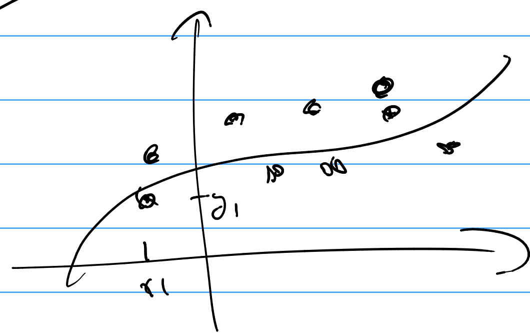
shift from $\mathbb{A}\mathbb{X} = \mathbb{b}$ (has no soln)

to $A^T A \mathbb{X} = A^T \mathbb{b}$ has a soln.

least squares solution

$\mathbb{X} = (A^T A)^{-1} (A^T \mathbb{b})$ is soln gets "closest" to $\mathbb{b}$.

ex

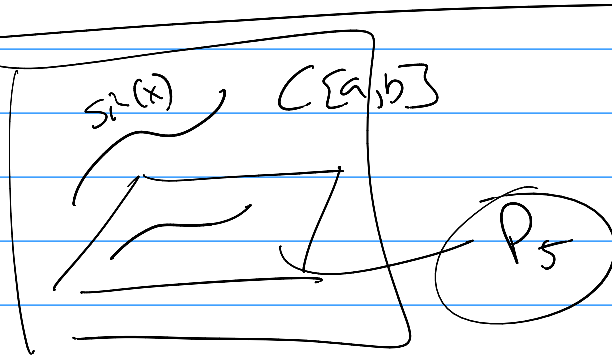


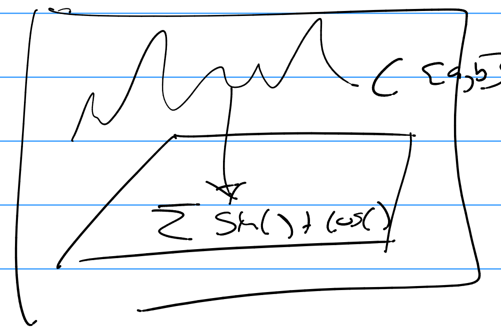
Data fitting

9 points fit it with

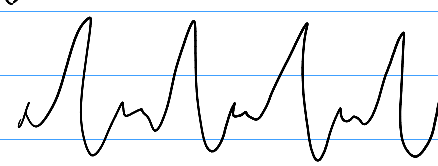
$f(x) = a + bx + cx^2 + dx^3$

$\mathbb{R}^n$

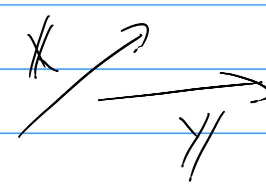




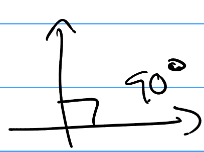
$\alpha_1 \sin(x) + \alpha_2 \cos x + \alpha_3 \sin(2x) + \alpha_4 \cos(2x)$



Chapter 5 Orthogonal



5.1

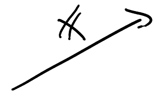


(1) $\mathbb{R}^2$ or $\mathbb{R}^3$ (Euclidean Geometry)

(2) Scalar Product

(row vector)(col. vector) = scalar

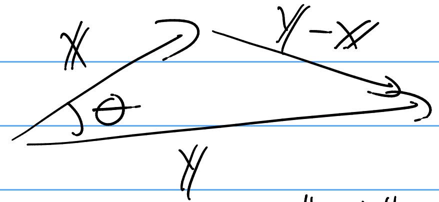
$\mathbb{X}^T \mathbb{Y} = x_1 y_1 + x_2 y_2 + \dots + x_n y_n$

Note: in **2D** 

length $x = \|x\| = (x_1^2 + x_2^2)^{1/2}$

in **3D** length $x = \|x\| = (x_1^2 + x_2^2 + x_3^2)^{1/2}$

in $\mathbb{R}^2, \mathbb{R}^3$ $\quad \|x\| = (x^T x)^{1/2}$ $\quad$ (**Notation:** $\|x\|^2 = x^T x$)

(3) 

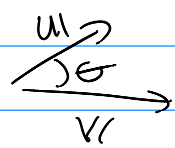
$$\|y - x\|^2 = \|x\|^2 + \|y\|^2 - 2\|x\|\|y\|\cos\theta$$

$$\begin{aligned} 2\|x\|\|y\|\cos\theta &= \|x\|^2 + \|y\|^2 - \|y - x\|^2 \\ &= x^T x + y^T y - [(y-x)^T (y-x)] \\ &= 2x^T y \end{aligned}$$

$\boxed{\text{Thm}}$ $\quad x^T y = \|x\|\|y\|\cos\theta$

or $\quad \cos\theta = \dfrac{x^T y}{\|x\|\|y\|}$

or if unit vectors u, v $\quad \cos\theta = u^T v$

so 

$\cos\theta = u^T v$

Facts: $u^T v = 0$ says $\cos\theta = 0$ $\quad$ $\perp$ orthogonal

$|u^T v| = 1$ says parallel.

abs. value

Def: $x^T y = 0$ call $x \perp y$ (orthogonal)

$\boxed{\text{Th}^n}$ Cauchy-Schwarz Inequality $(\mathbb{R}^2, \mathbb{R}^3)$

from: $\cos\theta = \dfrac{x^T y}{\|x\| \|y\|}$ and $-1 \le \cos\theta \le 1$

$0 \le |\cos\theta| \le 1$

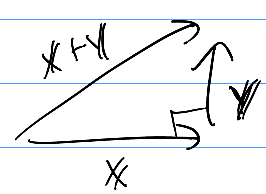
so $0 \le \left| \dfrac{x^T y}{\|x\|\|y\|} \right| \le 1$

$$0 \le |x^T y| \le \|x\|\|y\|$$

Applications:

① works in $\mathbb{R}^n$ (leave euclidean geometry)

use $x^T y = 0$ to be $x \perp y$

Note: 

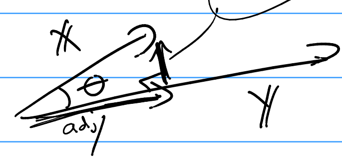
$\|x\|^2 + \|y\|^2 = \|x+y\|^2$

only if $x \perp y$

(means $x^T y = 0$)

② Projection: $x - p$ — part of $x \perp y$

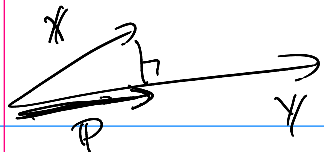
find p



part of x in direction of y = projection of x onto y = p

a) (by trig) $\cos\theta = \dfrac{\|adj\|}{\|x\|} = \dfrac{\|p\|}{\|x\|}$

$$\|p\| = \|x\| \cos\theta = \cancel{\|x\|} \frac{x^T y}{\cancel{\|x\|}\|y\|}$$



a) $\|p\| = \boxed{\dfrac{x^T y}{\|y\|} = \alpha \,:\, \text{"}\alpha \text{ is scalar proj. of } x \text{ onto } y\text{"}}$

b) $p = ? = \alpha \left(\dfrac{y}{\|y\|}\right) = \dfrac{x^T y}{\|y\|}\,\dfrac{y}{\|y\|} = \boxed{\dfrac{x^T y}{y^T y}\, y}$

unit vector in direction of y

"p is vector proj. of x onto y"

in use: