

Math 511

Pre-Req. arithmetic to the Calculus

Linear Algebra

Math = toys \oplus rules!

Linear Algebra

Solve:
$$\left. \begin{array}{l} 2x - y = 1 \\ x + y = 3 \end{array} \right\} \text{System of linear equations}$$

2 eqns and 2 unknowns

$$\left[\begin{array}{l} 2x - y + z = 1 \\ x + y - 3z = 2 \end{array} \right. \quad \text{2 eqns 3 unknowns}$$

$$\left[\begin{array}{l} x - y = 1 \\ x + y = 2 \\ 2x + 3y = 4 \end{array} \right. \quad \text{3 eqns and 2 unknowns}$$

Solve?

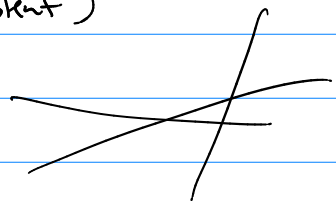
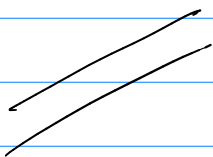
(1) Substitution

(2) elimination (add linear combos of eqns to get new eqns)

→ Possible Solutions:

(i) No solution (inconsistent)

(ii) 3 eqns, 2 unknowns



Systems of eqns

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m \end{cases} \quad \begin{array}{l} M \text{ eqns} \\ n \text{ unknowns} \end{array}$$

$$\downarrow$$
$$\left[\begin{array}{cccc|c} a_{11} & a_{12} & \dots & a_{1n} & b_1 \\ a_{21} & a_{22} & & a_{2n} & b_2 \\ \vdots & & & & \vdots \\ a_{m1} & a_{m2} & & a_{mn} & b_m \end{array} \right] \quad \text{augmented matrix.}$$

$$\textcircled{a} \quad \begin{cases} 2x - y + 3z = 1 \\ x + y - z = 2 \\ x + y = 1 \end{cases} \rightarrow \begin{array}{ccc|c} & x & y & z \\ \hline 2 & -1 & 3 & 1 \\ 1 & 1 & -1 & 2 \\ 1 & 1 & 0 & 1 \end{array}$$

Solve use elimination (elementary row ops)

① Swap rows.

② mult. row by non-zero

$$\textcircled{ex} \quad \begin{cases} x + \frac{1}{2}y = 2 \\ \uparrow \times 2 \\ 2x + y = 4 \end{cases}$$

③ linear combo of rows to get a new row.

Sol: to get an aug. matrix that is in triangular form.

\uparrow ex

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 1 & 3 \end{array} \right] \quad \begin{array}{l} x \quad y \quad z \end{array}$$

$$z = 3$$

$$y - x = 2$$

$$y = 5$$

$$x = -15$$